Control of a Ball on Sphere System with Adaptive Feedback Linearization method for regulation purpose

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ABSTRACT
This paper is the nonlinear adaptive feedback linearization control of a class of multi-input multi-output (MIMO) nonlinear systems called “system of ball on a sphere”, which is designed to control and operate a ball on the top of a sphere. Nowadays, Control of nonlinear systems using the feedback linearization has attracted lots of attention in the nonlinear control theory. Since in the general case, there is uncertainty with respect of these nonlinear systems parameters, Adaptive feedback linearization are employed to obtain asymptotically accurate cancellation for this inherent uncertainty. The system’s dynamic is described and the equations are illustrated. The results are simulated and compared in toe directions. The outputs are shown in different figures so as to be compared. These simulation results show the exactness of the controller’s performance.

KEYWORDS: Feedback linearization; Ball on a sphere; Adaptive; MIMO

1. INTRODUCTION
Recently, several attempts have been made to analyze the dynamic and of a system contain a ball on a body and its stability. This paper investigates a system of ball on a sphere which is particularly a nonlinear system. This system of ball on a sphere” is visualized in fig.1. In this paper, based on the results, a considerably simpler adaptive control technique for a larger class of these nonlinear systems is proposed. The applicability of feedback linearization, however, is somewhat limited due to the requirement of detailed knowledge of the system model [1]. In proportion to the availability of the measurement of system states, feedback linearization can be categorized into full-state feedback linearization and input-output feedback linearization [2]. The most important limitation of feedback linearization is that not all nonlinear systems are feedback linearizable [2]. In this paper, the adaptive feedback linearization method had been used. Adaptive feedback linearization method, as a famous controlling method, has already been used in several cases before [3-5]. In this articles case, the solution is given for the regulation case; when the derivative of the desired trajectory is known (measured).

Fig. 1. a ball on a sphere system

The reminder of this paper is organized as follows. In 2nd section, which is the dynamic and modeling section, the dynamic of the model has been presented and its parameters have been nominated. In the following, in the 3rd section, the control law has been described and by means of input-to-state stability theory; a new robust adaptive control scheme is designed involving the equations parameters. The simulation results have been discussed by the graphs in the 4th section and eventually the conclusion is presented in the 5th section.
2. DYNAMIC & MODELING

In this study, a ball on a sphere system with arbitrary desires is controlled by the robust adaptive controller. For this purpose, a model for the ball on a sphere system has been opted and then, its dynamical equations have been derived [6], [7]. Although these dynamical equations are extremely non-linear and their parameters are interdependent in various directions, they have been considered linear around the equilibrium point, since in that point, the parameters are assumed independent in all directions. In this case, the system of ball on sphere is considered to be tow dimensional in all directions, like a ball and wheel system.

The system parameters are $\theta_x$, $\theta_y$, which respectively denote the ball and the spheres angles with respect to the x direction, $\beta_x$ , $\beta_y$ which denotes the ball and the spheres angles with respect to the Y direction, $I_B$ and $I_b$ which are moments of inertia of the sphere and ball, respectively and $m$ as the balls mass. There is also $R$ and $r$ which already denote the sphere and balls’ radiuses respectively.

![Fig. 2. 2-D schema of the system](image)

Then, by using the Lagrangian method, the systems equation will be derived:

$$ L = K - U $$

$$ \frac{\partial}{\partial t} \left( \frac{\partial L}{\partial \dot{q}_i} \right) - \frac{\partial L}{\partial q_i} = Q_i, \, i = 1,2,3,4 $$

$$ Q_1 = 0 $$

$$ Q_2 = 0 $$

$$ Q_3 = T_x $$

$$ Q_4 = T_y $$

$$ (R + r)m + lb \frac{R + r}{r^2} \dot{\theta}_x + ( -lb \frac{R}{r^2} ) \dot{\beta}_x = 0 $$

$$ -mg \sin(\theta_x) = 0 $$

$$ (R + r)m + lb \frac{R + r}{r^2} \dot{\beta}_x + (-lb \frac{R}{r^2} ) \dot{\beta}_y = T_x $$

$$ (R + r)m + lb \frac{R + r}{r^2} \dot{\theta}_y + (-lb \frac{R}{r^2} ) \dot{\beta}_y = T_y $$

$$ -mg \sin(\theta_y) = 0 $$

$$ \dot{\theta}_y \left( R + r \right) \frac{R + r}{r^2} \dot{\beta}_x = T_x $$

$$ \dot{\beta}_y \left( R + r \right) \frac{R + r}{r^2} \dot{\beta}_y = T_y $$

$$ q = [\theta_x \beta_x \theta_y \beta_y] $$

$$ M \ddot{q} + G = T $$

$$ M = \begin{bmatrix} M_{11} & M_{12} & M_{13} & M_{14} \\ M_{21} & M_{22} & M_{23} & M_{24} \\ M_{31} & M_{32} & M_{33} & M_{34} \\ M_{41} & M_{42} & M_{43} & M_{44} \end{bmatrix} $$

$$ M_{11} = (R + r)m + lb \frac{R + r}{r^2} $$

$$ M_{12} = -lb \frac{R}{r^2} $$

$$ M_{13} = 0 $$

$$ M_{14} = 0 $$

$$ M_{21} = -lb \frac{R(R + r)}{r^2} $$

$$ M_{22} = IB + lb \frac{R^2}{r^2} $$

$$ M_{23} = 0 $$

$$ M_{24} = 0 $$

$$ M_{31} = 0 $$

$$ M_{32} = 0 $$

$$ M_{33} = (R + r)m + lb \frac{R + r}{r^2} $$

$$ M_{34} = -lb \frac{R}{r^2} $$

$$ M_{41} = 0 $$

$$ M_{42} = 0 $$

$$ M_{43} = -lb \frac{R(R + r)}{r^2} $$

$$ M_{44} = IB + lb \frac{R^2}{r^2} $$

$$ G = \begin{bmatrix} -mg \sin(q_1) \\ 0 \\ -mg \sin(q_3) \\ 0 \end{bmatrix} $$

$$ T = \begin{bmatrix} T_x \\ 0 \\ T_y \end{bmatrix} $$

So
\[ \dot{q} = M^{-1}(T - G) \]

For state space we have:

\[ \begin{align*}
    x_1 &= \theta_x \\
    x_2 &= \dot{\theta}_x \\
    x_3 &= \beta_x \\
    x_4 &= \dot{\beta}_x \\
    x_5 &= \theta_y \\
    x_6 &= \dot{\theta}_y \\
    x_7 &= \beta_y \\
    x_8 &= \dot{\beta}_y \\
\end{align*} \]

\[ \begin{align*}
    \dot{x}_1 &= x_2 \\
    \dot{x}_2 &= \ddot{q}_1 \\
    \dot{x}_3 &= x_4 \\
    \dot{x}_4 &= \ddot{q}_2 \\
    \dot{x}_5 &= x_6 \\
    \dot{x}_6 &= \ddot{q}_3 \\
    \dot{x}_7 &= x_8 \\
    \dot{x}_8 &= \ddot{q}_4 \\
\end{align*} \]

\[ \begin{align*}
    a_{01} &= (R + r)m \\
    a_{02} &= lb \left( \frac{R}{r^2} \right) \\
    a_{03} &= lb \left( \frac{R^2}{r^2} \right) \\
    a_{04} &= lb \\
    a_{05} &= \frac{lb}{r} \\
    a_{06} &= lb \frac{R}{r} \\
    a_{07} &= m \\
\end{align*} \]

\[ \rho = \begin{bmatrix}
    a_{01} \\
    a_{02} \\
    a_{03} \\
    a_{04} \\
    a_{05} \\
    a_{06} \\
    a_{07}
\end{bmatrix} \]

As a result:

\[ M = \begin{bmatrix}
    a_{01} + a_{02} + a_{05} & -a_{02} & 0 \\
    -a_{03} - a_{06} & a_{04} + a_{03} & 0 \\
    0 & 0 & a_{01} + a_{02} + a_{05}
\end{bmatrix} \]

\[ G = \begin{bmatrix}
    -a_{01} & 0 & 0 \\
    -a_{01} & 0 & 0 \\
\end{bmatrix} \]

\[ W = \begin{bmatrix}
    \dot{\theta}_x & \dot{\beta}_x - \beta_x & 0 & 0 & \ddot{\theta}_x & 0 \\
    0 & 0 & \dot{\beta}_x - \beta_x & \beta_x & 0 & -\ddot{\beta}_x - gsi \\
    0 & 0 & \ddot{\beta}_y - \beta_y & \dot{\beta}_y & 0 & 0 \\
    0 & 0 & \ddot{\beta}_y - \beta_y & \dot{\beta}_y & 0 & -\ddot{\beta}_y - gsi
\end{bmatrix} \]

As a result:

\[ M\ddot{q} + G = W\rho \]

3. ADAPTIVE FEEDBACK LINEARIZATION CONTROL

There are many different methods for controlling the robot manipulators, one of the model based controllers which is widely used is feedback linearization [8, 9]. The feedback linearization method, which was developed in [10], is utilized to design a nonlinear controller that will stabilize the nonlinear system expressed in section 2. Due to its precise and elegant theoretical foundations, feedback linearization has been used in many applications including robotic control [11]. This method is based on the exact model. A way to eliminate this difficulty is using adaptive feedback linearization in which the parameters are updated by a suitable update law that guarantees asymptotic stability.

The control law for this type of controller is [8]:

\[ \tau = \hat{M}(\theta) \left[ \ddot{\theta}_q - K_\theta \ddot{\theta}_q - K_\beta \ddot{\beta}_q \right] + \hat{C}(\theta, \dot{\theta}) \dot{\theta} + \hat{g}(\theta) \]

Where \( \hat{M}(q), \hat{C}(q, \dot{q}) \) and \( \hat{g}(q) \) are the estimate of the \( M(\theta), C(\theta, \dot{\theta}) \) and \( g(\theta) \) and \( \ddot{\theta} = \theta - \theta_q \), \( \dot{\theta} = \dot{\theta}_q \).

Considering \( \tau \) in the linear in parameters form one can find:

\[ \tau = Y(\theta, \dot{\theta}, \ddot{\theta}) \dot{\rho} \]

Where \( \rho \) is the parameter estimate vector, which is updated as below:

\[ \dot{\rho} = -\Gamma^{-1} \Phi^T B^T P \zeta \]

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All the parameters in the above equation can be found in [3], [4]. The control law (11) with the adaption algorithm makes it possible that the robot tracked the desired path asymptotically. The control law is shown graphically by Fig 3.

![Fig. 3. Adaptive feedback algorithm](image)

4. SIMULATION RESULTS

In order to have a regulation control for this system of “ball on a sphere”, the key parameters the ball and the sphere’s physical properties, which already described in the modeling section. The desired values of these parameters are listed in table 1 as below:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(m)</td>
<td>0.06 kg</td>
</tr>
<tr>
<td>(r)</td>
<td>0.0125 m</td>
</tr>
<tr>
<td>(R)</td>
<td>0.15 m</td>
</tr>
<tr>
<td>(I_B)</td>
<td>3.75×10^{-6} kg m^2</td>
</tr>
<tr>
<td>(I_B)</td>
<td>0.99 kg m^2</td>
</tr>
<tr>
<td>(g)</td>
<td>9.81 m/s^2</td>
</tr>
</tbody>
</table>

There are also desired values for the initial condition which are shown in table 2.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\theta_{x0})</td>
<td>0.07</td>
</tr>
<tr>
<td>(\theta_{x0})</td>
<td>0.02</td>
</tr>
<tr>
<td>(\beta_{x0})</td>
<td>0</td>
</tr>
<tr>
<td>(\beta_{x0})</td>
<td>0</td>
</tr>
<tr>
<td>(\theta_{y0})</td>
<td>0.07</td>
</tr>
<tr>
<td>(\theta_{y0})</td>
<td>0.05</td>
</tr>
<tr>
<td>(\beta_{y0})</td>
<td>0</td>
</tr>
<tr>
<td>(\beta_{y0})</td>
<td>0</td>
</tr>
</tbody>
</table>

The controlling simulation has been done due to the control law shown in Fig. 3, the initial parameters and system properties. These simulation results are summarized in Figs 5-13.
5. CONCLUSION
The purpose of this article was to control a system of “ball on a sphere”. To gain this goal, a dynamical model that had been constructed for this special system, had been used. As was evidence in the simulation results and figures, the adaptive feedback linearization controller was perfectly able to control this system with uncertain parameters and was already set to lead the system to the desired position. The great accuracy of the diagrams represents the used controllers controlling power in which can continue doing its task well.

REFERENCES