Identification of a Two Degree of Freedom Tracker System: Theoretical and Experimental Discussion

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ABSTRACT: In this article, the identification problem of a practical two degree-of-freedom (DOF) tracker system is investigated. The analytical modeling of two-DOF gimbal system as the main core of the tracker for both elevation and azimuth axes are introduced. By simplification and discussion on the governing equations, suitable structure of the model for identification is obtained. By performing identification procedure on the experimental system in the elevation and azimuth axes, the simplified models of this system are obtained through Recursive Least Square (RLS) approach. This is performed by using the practical data obtained from gyro in each axis of the under study system. As it can be interpreted and predicted from theoretical model, the identification process leads to a non-minimum phase model which depends on the operational points. This identified model can be used in stabilization and tracking loops in the control system design stage. The identification results in comparison with theoretical and experimental data are discussed. The adaptive or robust controllers are suitable candidates for controlling tracker system, where in the both controllers use this simplified model as the nominal or reference model.

KEYWORDS: Two-DOF tracker system, Identification, Recursive Least Square (RLS), Line-of-Sight (LOS)

1. INTRODUCTION

Tracker systems are used in many guidance systems where a sensor should lock on a fixed or moving target with desired precision to obtain the target position in 3D space. The advances made in industry, appearance of new vehicles, exact control, navigation system, new generation of missiles production, spacecraft and defense industry lead researches to apply tracking of fixed and movable targets. To have desired performance of tracking in spite of system movements and external disturbances, the sensors should be isolated from the system main body. For this purpose, two-DOF gimbal system is used as the main mechanical part in these systems. Other main parts of a tracker system are: gyros, motors, gears, and image processing units. In fact, the purpose of controlling and guiding system is to keep specified line-of-sight (LOS) in 3D space and tracking the target. Hence, equipment is installed on stable platform.

Two-DOF gimbal system is usually adopted to provide stability for the LOS when different disturbance terms effects on it. The important sources of disturbance in this system are: the angular position of gimbal systems, friction between frames resulted of system dynamic, a cable restraint and mass unbalance. The mass unbalance is a serious and inevitable problem in the gimbal system even if the controller is designed correctly. It is necessary for all dynamics of the system to be considered and the system must be analytically analyzed before design and production. The mathematical model of two DOF gimbal system are considered in here. It is clear that the system performance depend on accuracy of modeling extremely.

A comprehensive model of two DOF gimbal system based on dynamic equation is introduced in [1]. In [2], the mass unbalance is surveyed for this system. The effect of cross coupling on two DOF gimbal system with unbalanced dynamics is considered in [3]. In [4],
the gimbal equations are obtained from the Lagrange relation by analyzing the angular position. In [5], the model of servo controlled system for gimbal system around a free axis by using cascade PID controller is presented. A nonlinear model, in spite of uncertainty for inertial space platform (ISP) is offered in [6]. Thus, Cable restraint, friction, body movement on the system and other disturbances in environment, are discussed in this model. It is possible to overlap the gravity center on rotating center by adding the outer unbalance mass to gimbal system and increase the tracker precision and the stabilizer ability [7]. In [8], the cross coupling kinetic relations for two-DOF gimbal are considered when each gimbal is stabilized and the gimbal body is suspending around the main axes. In [9], the inertial stabilized platform is surveyed as a good solution for this problem. In [10], the dynamic equation is studied with assumption that gimbal has no mass unbalance. The disturbance torques has been omitted by symmetry in the inertia matrix in [11]. The dynamic model of three-axis gyro stabilized platform extracted from Newton Euler in [12]. A simple dynamic model and applicable is exhibited for designing a robust control system in [13]. In [14], the uncertainty is considered on the system parameters in the disturbance terms.

In none of the reviewed articles, a simplified model for two-DOF gimbal systems is not calculated where it is required to serve reference model for adaptive or robust controller. In addition, the existence of uncertainty in the system disturbances are augmented its demand. Thus, in this article, a simplified model of two-DOF gimbal system is extracted from the practical data. This contribution is important because the system model is identified through the practical data on the experimental system in the elevation and azimuth axes. The identification results in comparison with theoretical and experimental data are discussed in this article.

For this purpose, the analytical model of two-DOF gimbal system is explained in section 2. The movement equations are exploited around the azimuth and elevation axes. After introduction of the gimbal system equations, in section 3, the analysis of system identification from practical data through RLS approach is carried out. In section 4, the numerical results of gimbal system are provided.

2. **Analytical modeling of two-DOF gimbal System**

Gimbal system is a two input-output system with complicated nonlinear relations used in autonomous navigation systems for guiding and controlling in aerospace, astronavigation, missiles and ships. Two-DOF gimbal system is composed of two subsystems named inner gimbal and outer gimbal. A general schematic of this system has been shown in Fig. 1.
2.1. Equation of gimbal system movement

In order to analyze the equation of gimbal system movement, three reference frames are described here. For each one of the system frames at every side, special angular velocity is defined as:

\[
\omega_b = \begin{bmatrix} p \\ q \\ r \end{bmatrix}, \quad \omega_i = \begin{bmatrix} p_i \\ q_i \\ r_i \end{bmatrix}, \quad \omega_o = \begin{bmatrix} p_o \\ q_o \\ r_o \end{bmatrix}
\]

Where, \( \omega_b \) is the vector of gimbal body velocity, \( \omega_i \) is the vector of inner frame velocity, and \( \omega_o \) is the vector of outer frame velocity. These three frames which overlap are centered on the rotating center assuming that the gimbal has no mass unbalanced. These two systems are obtained by two transformation matrixes in relation to coordinates of the system body as below:

\[
L_{ob} = \begin{bmatrix} \cos \phi & \sin \phi & 0 \\ -\sin \phi & \cos \phi & 0 \\ 0 & 0 & 1 \end{bmatrix}
\]

\[
L_{io} = \begin{bmatrix} \cos \theta & 0 & -\sin \theta \\ 0 & 1 & 0 \\ \sin \theta & 0 & \cos \theta \end{bmatrix}
\]

(2) (3)

where, \( L_{ob} \) is the transformation matrix from body to outer frame and \( L_{io} \) is the transformation matrix from outer frame to inner frame. The inertial momentum matrixes in inner and outer gimbal are describes as follows:

\[
J_i = \begin{bmatrix} J_{ix} & D_{ix} & D_{ix} \\ D_{ix} & J_{iy} & D_{iy} \\ D_{ix} & D_{iy} & J_{iz} \end{bmatrix}
\]

\[
J_o = \begin{bmatrix} J_{ox} & d_{ox} & d_{ox} \\ d_{ox} & J_{oy} & d_{oy} \\ d_{ox} & d_{oy} & J_{oz} \end{bmatrix}
\]

(4) (5)

where \( J \) is the inertial momentum and \( d \) is the inertial products for outer and \( D \) is the inner gimbal. Thus, \( q_i \) and \( r_i \) are measured by the rate gyro which is mounted on inner gimbal. Hence, the following two vectors are considered as:

\[
\phi = \begin{bmatrix} 0 \\ 0 \\ \phi \end{bmatrix}
\]

(6)

By using the above transformation, the angular velocities between inner gimbal and body as well as inner and outer gimbal are presented as follows:

\[
\omega_o = L_{ob} \omega_b + \phi
\]

\[
\omega_i = L_{io} \omega_o + \phi
\]

(8) (9)

Thus, the angular velocities of the inner and outer gimbals are calculated by the following equations:

\[
p_i = p_o \cos \phi + q_o \sin \phi
\]

\[
q_i = q_o + \dot{\phi}
\]

\[
r_i = r_o \sin \phi + r_o \cos \phi
\]

(10) (11)

2.2 The gimbal system movement elevation axis equation

The disturbance torques will be added to the model based on the movement relation in every gimbal system. As \( T_{el} \) is the controlling torque and \( q_i \) is the inner angular velocity of elevation axis. Thus, the dynamic equation around elevation axis, with assumption that system is balancing, is presented as:

\[
J_i \ddot{q}_i = T_{el} + T_{d-el}
\]

(12)

The torque \( T_{d-el} \) which is the undesirable disturbance calculated as follow:

\[
T_{d-el} = (J_{iz} - J_{ix}) p_i r_i + D_{ix} (p_i^2 - r_i^2) - D_{xz} (r_i - p_i q_i) - D_{xy} (p_i + q_i r_i)
\]

(13)

The block diagram of the seeker system in elevation axis is shown in Fig. 3.

Fig. 3: The block diagram of torque-velocity in elevation axis

2.3 The gimbal system movement azimuth axis equation
By using the momentum equation, the gimbal system equations in azimuth axis are calculated as follows:

\[ J_o \ddot{\varphi} = T_{d1} + T_{d2} + T_{d3} \]

(14)

Where \( T_{d1}, T_{d2}, \) and \( T_{d3} \) are undesirable disturbances.

\[ T_{d1} = [J_{oz} + J_{ix}] \sin^2 \vartheta + J_{iz} \sin^2 \vartheta + D_{xz} \sin 2\theta - (J_{oy} + J_{iy})p_0 q_o \]

\[ T_{d2} = -[d_{xz} + (J_{iz} - J_{ix}) \sin \theta \cos \theta + D_{xz} \cos 2\theta] \]

(15)

\[ (p_o - q_o) - (d_{yz} + D_{yz} \cos \theta - D_{xy} \sin \theta) \]

(16)

\[ (d_o + p_o q_o) - (d_{xy} + D_{xy} \cos \theta + D_{yz} \sin \theta)(p_o - q_o) \]

\[ T_{d3} = \theta(D_{xy} \sin \theta - D_{yz} \cos \theta) + \theta(J_{ix} - J_{iz}) \]

(17)

Concerning the mentioned statements, the total diagram schematic of azimuth axis equations is present in the following diagram:

Fig. 4: The block diagram of torque-velocity in azimuth axes

3. Identification of current experimental system

This system can rotate on two axes of elevation and azimuth, with the two gyros installed on each other. In fact, these gyros are applied to get feedback from angular velocities. To have a good identification, the input of identification system must have the same frequency spectrum in all band width. As a matter of fact the signal of input control must be able to excite the different modes of the system. On the other hand, the system motion equation for the obtained parameters from RLS approach can explain the nonlinear manner of the system in all favorable operational points.

3.1 Extraction of suitable model structure for identification

One of the best manners for modeling is to apply the dynamic equations of any system. In fact identification is necessary where to exploit the voltage parameters of this model by simplifying the obtained equations in trackers system, the structure of a suitable model is extracted for identification. The identified model order for elevation and azimuth axes have been selected as regards the dynamic equation of two-DOF gimbal system. Also, a zero is considered in the transfer function as respect to response of the rate gyro output and the disturbance effects in these systems. The transfer function between input voltage of drive system \((V_{in})\) and output angular velocity \((q)\) in elevation and azimuth axes with regardless the total disturbance term and the assumption that the fixed tracker body is simplified as follow:

\[
\begin{align*}
q_1(z) &= \beta_1 + \beta_2 z^{-1} \\
q_2(z) &= \beta_3 + \beta_4 z^{-1} \\
V_{in} &= 1 + \alpha_1 z^{-1} + \alpha_2 z^{-2} \\
V_{in} &= 1 + \alpha_3 z^{-1} + \alpha_4 z^{-2}
\end{align*}
\]

(19)

As observes, all \( \alpha \) and \( \beta \) parameters applied in the structure of this model depend on the creating angle either in direct manner. Thus, the change range of the mentioned parameters in different application points is vast.

3.2 Experimental

After determining the structure of the suitable model, the equation of experiment plant in the course of identification will be specified. In order to collect the essential input-output information for identification of system parameters by RLS approach, some experiments are running by applying some signals as the input voltage of torque generator. Moreover the output angular velocity will be measured and recorded through special gyro installed in the inner frame of the gimbal. These experiments must be performed for different inner angles of every frame in relation with another frame.

3.3 Calculation of the applied parameters in structure of the model

To calculate the different parameters in elevation and azimuth axes the algorithm of data processing is considered by the RLS approach. States based on the fact that the summation square difference between output model and real value is low where some weight factors for measuring precision are applied. The range of parameters changes in the elevation axis dose not highly depend on the angle of azimuth axis, while the parameters change in the parameters azimuth is highly depend on angular changes of elevation axis. This issue can clearly demonstrate a reduction in azimuth axis loop gain by an increase in the angle of
system elevation axis.

3.4 Determination the model validation

For model validation test, the identified systems for azimuth and elevation axes have been applied for some of experimental data. The obtained model from the experimental tracker system indicates that the RLS approach is acceptable and corresponds to the available findings and compatible.

4. Numerical results

4.1 The numerical results of gimbal system in elevation axis

The actual responses and identified system to step input are shown in the Fig. 5 where both the responses are in complete overlap. Thus, for model validity, the responses with test data are analyzed for elevation axis in Fig. 6.

![Fig. 5: The system identification of elevation axis.](image)

![Fig. 6: The model validation in elevation axis.](image)

The norm of identified error in elevation axis in Fig. 7 demonstrates that the identified parameters, regarding the practical aspect of signals, are in acceptable error range. Also, the parameters estimation is shown in Fig. 8.

![Fig. 7: The error response in elevation axis.](image)

![Fig. 8: The parameters estimation.](image)

4.2 The numerical results of gimbal system in azimuth axis

The simulation results of gimbal system azimuth axis similar elevation axis are simulated.

![Fig. 9: The system identification of azimuth axis.](image)

![Fig. 10: The model validation in azimuth axis.](image)

The transfer function of identified system in azimuth and elevation axes is obtained respectively in equation (20) and (21).

\[
q_{f}(z) = 0.008145 + 0.008145z^{-1}
\]

\[
v_{in}(z) = -1.514z^{-1} + 0.5296z^{-2}
\]

(20)
The gimbal system around the elevation and azimuth axes has a zero on unit circle which increase the instability condition where the disturbance terms is applied to system. In addition, analyze in the identification results indicates that the position of the elevation axis has more effect on the azimuth axis in the LOS control system. This issue shows that the unsuitable performance of elevation axis distorts completely the azimuth axis position. This identified model can be used in feature articles in stabilization and tracking loops at the control system design stage. With respect to the proposed structure and that it fact system is non min-phase its contribution toward the issue of control design studies is available.

5. Conclusion
In this article, two axes gimbal systems are analyzed. The purpose here is to obtain an identified model for gimbal system and the features of its structure, and obtain a proper model for analyzing the method of control design. For design and implementation of each controller based on motion equations, recognizing the system parameters like inertial moment, friction factors and the fixed torque are necessary. Determining some of these parameters is very difficult for experimental systems and some precise measurement devices are required. It is possible to determine these parameters by some other manners approaches like RLS. The responses of the main system and identification system to step input are presented in the elevation and azimuth axes in a row. As observed, the amplitude of the identified system responses and the main system are overlap. This proposed approach is measured and recorded, followed by an approximation of system parameters calculates through some identification experiments that this identification system could be adopted in designing model base controller.

6. REFERENCES