Synchronization of Hyperchaotic Systems with Integral Active Sliding Mode Control

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ABSTRACT:
In this paper present a new surface in an active sliding mode to synchronize two chaotic systems. To verify the capability of the proposed scheme, signals are also contaminated by measurement noise. The integral acting surface produces a dynamics for error, where the appropriate eigenvalues are easily assigned. Using this surface, calculation of parameters of the controller becomes simpler than the classical alternative. A sufficient condition, as a guideline of the designated procedure, is dedicated to provide a robust stability of the error dynamics. Finally, a simulation study is performed to verify the robustness and effectiveness of the proposed control strategy.

KEYWORDS: Component, Chaos Synchronization, Hyperchaos, Integral; Active Sliding Mode Control, Hyperchaotic Chen System

1. INTRODUCTION
In recent years, the chaos theory and relevant properties have found useful applications in many engineering areas such as secure communication, biological systems, power electronic devices and power quality, digital communication, chemical reaction analysis, [1-11]. Since the pioneering work by Pecora and Carroll ([2], 1990), chaos synchronization problem has been studied extensively and intensively in the literature [2-17]. They reported that chaotic systems possess a self synchronization property.

In most of the chaos synchronization approaches, the master-slave or drive-response formalism is used. If a particular chaotic system is called the master or drive system and another chaotic system is called the slave or response system, then the goal of the chaos synchronization is to use the output of the master system to control the slave system so that the output of the slave system tracks the output of the master system asymptotically.

The design procedure of an active sliding mode controller is a combination of an active controller and a sliding mode one. During some active sliding mode design procedures, there are some parameters of the controller which are needed to be determined however, determination of the parameters is a somewhat exhaustive task. In this paper, a new surface designation will be introduced to cope with this problem.

The remainder of this paper is organized as follows. In Section 2, an active sliding mode control is introduced. In Section 3, we describe the theory of the active sliding mode and stability of the controller. In Section 4, we discuss Synchronization of same chaotic systems are given to illustrate the effectiveness of the proposed method. Finally, a conclusion in Section 5 closes the work.

2. SYSTEM DESCRIPTION AND THE PROBLEM FORMULATION
In this section, we detail the problem statement for chaos synchronization of identical chaotic. Consider the chaotic system described by

\[ \dot{x} = A_1 x + f_1(x) \]  

(1)

Where \( x \in \mathbb{R}^n \) is the state of the system, \( A \) is the \( n \times n \) matrix of the system parameters and \( f : \mathbb{R}^n \rightarrow \mathbb{R}^n \) the nonlinear part of the system. We consider the system (1) as the master or drive system. The nonlinear part of the system. We consider the system (1) as the master or drive system.

As the slave or response system, we consider the following chaotic system described by the dynamics
The synchronization problem is to design the controller $u(t)$ which synchronizes the states of the slave with that of the master. However, the synchronization goal is as follows:

$$\lim_{t \to \infty} \|e(t)\| = 0$$

Where $\| \|$ is the Euclidean norm (2-norm) of the vector.

To solve this problem, we first define the control input $u$ as

$$u(t) = H(t) - f_2(y) - f_1(x) - (A_2 - A_1)x$$

In which $H(t)$ is designed based on a sliding mode control law. Although, there are many possible choices for the control $H(t)$, without loss of generality, the sliding mode control law is chosen by:

$$H(t) = Kw(t)$$

Here $w(t) \in R$ is a control input and can be determined as:

$$w(t) = \begin{cases} w^+(t), s(e) > 0 \\ w^-(t), s(e) \leq 0 \end{cases}$$

Where $s = s(e)$ is a switching surface which introduces the desired dynamics.

Substituting (7) into (5), the error dynamics simplifies to

$$\dot{e} = K w(t) + A_2 e$$

In the sliding mode control, we define the variable

$$s(e) = e - (K + A_2) \int_0^t e(\tau) d\tau$$

When in sliding manifold $s$ the system (10) satisfies the following conditions:

$$s(e) = 0$$

Which is the defining equation for the manifold $s$ and

$$\dot{s}(e) = 0$$

Which is the necessary condition for the state trajectory $e = 0$ of (10) to stay on the sliding manifold $s$.

Using (10) and (11), the equation (13) can be rewritten as

$$\dot{s}(e) = e - (K + A_2)e = K(w - e) = 0$$

Solving (14) for $w$, we obtain the equivalent control law given by

$$w_{eq} = e$$

Substituting (15) into the error dynamics (10), we obtain the closed-loop dynamics as

$$\dot{e} = (K + A_2)e$$

$K$ is turned such that all eigenvalues of $K + A_2$ have negative real parts, hence the system is asymptotically stable. A constant plus proportional rate reaching law is a concern here.

Then the controlled system (16) is globally asymptotically stable.

To design the sliding mode controller for the linear time-invariant system (10), we use the constant plus proportional rate reaching law

$$\dot{s} = -q \text{sgn}(s) - rs$$

Where $\text{sgn}(.)$ denotes the sign function and the gains $q > 0, r > 0$ are determined so that the sliding condition is satisfied and sliding motion will occur.

From equations (14) and (17), we can obtain the control $w(t)$ as

$$w(t) = K^{-1}(-q \text{sgn}(s) - rs + Ke)$$

**Proof.** First, we note that substituting (16) and (18) into the error dynamics (5), we obtain the closed-loop error dynamics as

$$\dot{e} = (A_2 + K)e - \text{sgn}(s) - rs$$

**A. Robust stability analysis**

To prove that the error dynamics (19) is globally asymptotically stable, we consider the candidate Lyapunov function defined by the equation

$$V(e) = 1/2s^2(e)$$

Which is a positive definite function on $R^n$.

Differentiating $V$ along the trajectories of (20) or the equivalent dynamics (16), we get

$$\dot{V} = \dot{s} = -q \text{sgn}(s) - rs^2$$

This calculation shows that $V$ is a globally defined, positive definite, Lyapunov function for the error dynamics (19), which has a globally defined, negative definite time derivative $\dot{V}$.

Thus, by Lyapunov stability theory, it is immediate that the error dynamics (19) is globally asymptotically stable for all initial conditions $e(0) \in R^n$.

This means that for all initial conditions $e(0) \in R^n$, we have $\lim_{t \to \infty} \|e(t)\| = 0$.

Hence, it follows that the master system (1) and the slave system (2) are globally and asymptotically synchronized for all initial conditions $x(0), y(0) \in R^n$.

This completes the proof.
CHAOSSYNCHRONIZATIONOFIDENTICALHYPERCHAOTICCHENSYSTEMS

A. Main Results

In what follows, the active sliding mode control approach for synchronization of two chaotic systems will be simulated. A fourth order Runge–Kutta solver with time step size of 0.001 s is performed to solve the set of differential equations, concerning the master and slave system.

Thus, the master system is described by the hyperchaotic Chen dynamics

\[
\begin{align*}
\dot{x}_1 &= 35(x_2 - x_1) + x_4 \\
\dot{x}_2 &= 7x_1 - x_1x_3 + 12x_2 \\
\dot{x}_3 &= x_1x_2 - 3x_3 \\
\dot{x}_4 &= x_2x_3 + 0.5x_4
\end{align*}
\]

(22)

Where \(x_1, x_2, x_3, x_4\) are the states of the system.

The slave system is described by the controlled hyperchaotic Chen dynamics

\[
\begin{align*}
\dot{y}_1 &= 35(y_2 - y_1) + y_4 + u_1 \\
\dot{y}_2 &= 7y_1 - y_1y_3 + 12y_2 + u_2 \\
\dot{y}_3 &= y_1y_2 - 3y_3 + u_3 \\
\dot{y}_4 &= y_2y_3 + 0.5y_4 + u_4
\end{align*}
\]

(23)

Where \(y_1, y_2, y_3, y_4\) are the states of the system and \(u_1, u_2, u_3, u_4\) are the controllers to be designed.

The state of the hyperchaotic Chen system are shown in Fig. 1. The chaos synchronization error \(e\) is defined by

\[
e_i = y_i - x_i \quad i = (1,2,3,4)
\]

(24)

The error dynamics is easily obtained as

\[
\begin{align*}
\dot{e}_1 &= 35(e_2 - e_1) + e_4 + u_1 \\
\dot{e}_2 &= 7e_1 + 12e_2 - y_1y_3 + x_1x_3 + u_3 \\
\dot{e}_3 &= -3e_3 + y_1y_2 - x_1x_2 + u_4 \\
\dot{e}_4 &= 0.5e_4 + y_2y_3 - x_2x_3 + u_4
\end{align*}
\]

(25)

The matrix gain \(K\) is designed such that the real part of the error dynamics in (16) is kept negative. This surface easily defines the appropriate \(K\) for the active sliding mode. Meanwhile the rate of convergence of the error will be arbitrarily tuned by proper choice of the eigenvalues of dynamics in (16), despite of the traditional technique. As a case study the gain can be chosen as:

\[
K = \text{diag}([-10, -20, -2, -12])
\]

(26)

B. Numerical Results

For the numerical simulations, the fourth-order Runge Kutta method with time-step \(h = 10^{-3}\) is used to solve the hyperchaotic systems (22) and (23) with the sliding mode controller \(u\) given by (7) using MATLAB. The sliding mode gains are chosen as \(r = 5, q = 0.2\). The initial values of the master system (23) are taken as:

\[
x_1(0) = 30, x_2(0) = 16, x_3(0) = 18, x_4(0) = 20
\]

The initial values of the slave system (24) are taken as:

\[
y_1(0) = 12, y_2(0) = 22, y_3(0) = 30, y_4(0) = 6
\]

Fig. 2 depicts the complete synchronization of the identical hyperchaotic Chen systems (23) and (24).

5. Conclusion

In this paper, a novel control scheme has been proposed for chaos synchronization using a new sliding surface. It was shown that the prescribed surface can be selected such that designation of parameters of the controller becomes simpler. It was illustrated that the error asymptotically approaches zero. An integral action takes place in the sliding surface which causes the slave to track the master, faster than using traditional surfaces (\(s = Ce\) in which \(C\) is a constant vector) under the same conditions. Consequently, the surface dynamic is kept regulated at a constant. Numerical results verify the effectiveness and robustness of the proposed control approach.

REFERENCES


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Fig. 1. State Orbits of the Hyperchaotic Chen System
Fig. 2. Synchronization of the Identical Hyperchaotic Chen Systems