Performance of the Developed Versions of CFAR Schemes Processing Non-Coherently Integrated M-Pulses in the Presence of Outliers

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ABSTRACT:
The CFAR processors have been utilized in radar systems where the clutter environment is partially unknown and/or has varying statistical properties. In such situations, the performance of the fixed-threshold detector deteriorates significantly, and the CFAR detector is designed to be invariant to changes in the clutter intensity. The CA and OS procedures are the most popular ones in the CFAR world. The CA is the optimum processor in terms of detection probability in homogeneous background while the OS algorithm has its immunity against outlying targets whenever their number is within an allowable range. To benefit the merits of these two algorithms, new versions, which combine the basic of these techniques in estimating the unknown noise power level, have been recently proposed. These versions have improved detection performance either they operate in homogeneous or in non-homogeneous background environments. This paper is devoted to the detection analysis of these versions when the radar receiver contains a noncoherent integrator amongst its fundamental elements. Exact formulas for their false alarm and detection probabilities are derived, in the absence as well as in the presence of interferers. The primary as well as the secondary outlying targets is assumed to be fluctuating in accordance with SWII model. Our obtained results indicate that while the SO version performs nearly as good as the conventional OS scheme in the presence of outlying targets, all the developed versions perform much better than OS in homogeneous situations. In comparison with the CA detector, the modified versions perform better in an ideal operating environment, and behave much better when the background environment is contaminated with a number of spurious targets.

KEYWORDS: Adaptive Detection Algorithms, CA and OS Schemes, Modified Versions, Non-Coherent Integration, Fluctuating Targets, Multiple-Target Environments.

1. INTRODUCTION
Radar has become an essential component of current defensive systems. It has been used in a variety of military and civilian applications owing to its ability to survey wide areas rapidly during the day or at night and in all weather conditions. In other words, the radar is the only sensor that is capable of detecting aircraft out to ranges of hundreds of kilometers. Because of this, there are some networks of civil aviation radars that form a part of wider air defense capabilities. These networks are specifically designed to ensure early warning against potentially hostile threat targets. Additionally, imaging radars carried by aircraft or satellites are routinely able to achieve high-resolution images of target scenes, and to detect and classify stationary and moving targets at operational ranges. Short-range radar techniques may be used to identify small targets, even buried in the ground or hidden behind building walls [1].

The radar system is often required to detect targets against a changing background of clutter and thermal noise. A common routine test in any detection system is to compare the received signal level with a predefined threshold value. If the threshold is crossed, the presence of the signal of interest is declared. The standard detection strategy is to try and maintain a fixed probability of false alarm, whilst also attempting to maximize the probability of detection. To achieve this interesting requirement, the radar detection processing must be able to estimate the mean power of the interference, in order to set a reference level (threshold) against which the received is compared to decide if the searching target is absent or present. On the other hand, in modern radar detection, the decision on target presence or absence is often performed
automatically, that is, without the visual intervention of the radar operator. When the threshold is of a fixed value, the false alarm rate will increase intolerably (i.e., beyond a level that the computer of an automatic detector can handle) as the interference power varies. In general, the mean noise power is unknown and may also be varying temporally and spatially, requiring a continuously changing the detection threshold in order to follow these variations. For the radar system to be adapted to unknown clutter amplitude, it is required to maintain the received signal level within the dynamic range of the radar receiver, to prevent distortion of the signals. In addition, the local mean level of the interference (i.e. clutter-plus-noise) must be estimated for the detection threshold to be constructed. The process of continuously changing the threshold value to maintain a constant rate of false alarm is known as CFAR. The requirement of achieving CFAR process is central to the practical operation of many radar systems. The problem is how to estimate the interference power, in particular at the clutter boundaries (e.g. from land to see clutter) and for multiple targets, such that no target is suppressed. In a radar receiver, after amplitude detection, the backscattered signal is sampled in range and/or Doppler and a one- or two-dimensional reference window is formed. The detection in radar means existence or nonexistence of a target in the middle cell of a reference window or a cell under test. The noise and clutter background is estimated by processing the output from neighboring cells. The selection of the cells for CFAR threshold calculation and the type of averaging is important here. Mean and censored mean value, greatest-of, smallest-of, order-statistics, and trimming estimation procedures have been proposed. All of these have advantages and disadvantages none of these is globally optimum [2]-[12].

A variety of CFAR techniques are developed according to the logic used to estimate the unknown noise power level. An attractive class of such schemes includes CA, OS and their modified versions. The threshold in a CFAR detector is set on a cell basis using estimated noise power by processing a group of reference cells surrounding the cell under test. The CA processor is an adaptive scheme that can play an effective part in much noise and clutter environments, and provide nearly the best ability of signal detection while reserving the enough constant false alarm rate. This algorithm has the best performance in homogeneous background since it uses the maximum likelihood estimate of the noise power to set the adaptive threshold. However, the existence of heterogeneities in practical operating environments renders this processor ineffective [2], [7]. Heterogeneities arise due to the presence of multiple targets and clutter edges. In the case of multiple targets, the detection probability of CA degrades seriously due to the non-avoidance of including the interfering signal power in noise level estimate. Consequently, this in turn leads to an unnecessary increase in overall threshold. When a clutter edge is present in the reference window and the test cell contains a clutter sample, a significant increase in the false alarm rate results. Both of these effects worsen as the clutter power increases. In order to overcome the problems associated with non-homogeneous noise backgrounds, alternative schemes have been developed to address this issue, including order-statistics (OS) and its versions as well as various windowing techniques aimed to exclude heterogeneous regions. The well-known OS processor estimates the noise power simply by selecting the Kth largest cell in the reference set of size N. It suffers only minor degradation in detection probability and can resolves closely spaced targets effectively for K different from the maximum. However, this processor is unable to prevent excessive false alarm rate at clutter edges, unless K is very close to N, but in this case the processor suffers greater loss of detection performance [8, 13]. In practical applications, the information about the number of interfering targets is not known in advance. The biggest samples of reference window are always trimmed with OS or TM method not only in multiple targets situation but also in homogeneous background. This, in turn, will result in additional CFAR loss, especially in the case where the size of the reference window is short. The resulting CFAR loss becomes unacceptable, and this can usually be encountered in complicated environment and lower SNR situation [11], [15].

Two novel constant false alarm rate detectors; the maximum (MX) and the minimum (MN), are recently appear in the literature [3]. These new formulas of CFAR detectors improve the conventional CA and OS schemes by making full use of the cell information. The novel CFAR processors combine the result of the CA and OS to get a better detection performance. They make use of the two threshold settings of the well-known processors and compare them with the cell under test to achieve the judgment. For the MX-CFAR algorithm, when the cell under test is greater than both of the CA and OS thresholds, a target present will be declared. Otherwise, no target will be indicated. For the MN algorithm, on the other hand, when the cell under test is greater than any of the two thresholds, a target present will be declared. Otherwise a no target declaration will be made. These two new radar CFAR detectors synthesizing the advantages of CA and OS algorithms are analyzed in an ideal environment [3]. The non-homogeneous performance of these two new versions along with a new one is analyzed by El_Mashade [14]. Our goal in the present paper is to
analyze these hybrid versions in non-homogeneous situations when the radar receiver collects data from M pulses to carry out its task of detection. In section II, we formulate the underlined problem and describe the model of the processors under investigation. The performance of the schemes under consideration is analyzed when the operating environment is free of or contaminated with a number of spurious targets along with the target under searching in section III. Section IV is devoted to the performance assessment of these schemes in different operating conditions. In section V, we present a brief discussion along with our conclusions.

2. STATISTICAL BACKGROUND AND PROBLEM FORMULATION

The functional scheme of the CFAR detector is outlined in Fig.(1). The received IF signal is applied to a matched filter which is specifically designed to maximize the output signal-to-noise ratio (SNR). The output of the matched filter is then passed through a square-law device to extract the baseband signal. Following the square-law device, M consecutive sweeps are noncoherently integrated to form the input of the CFAR circuit. The integrator output is then sampled and the sampling rate is chosen in such a way that the successive samples are statistically independent. A set of N samples, called the sample set, is used for the noise level estimation. It is assumed that the sample tested for detection is excluded from this set and thus ensure that the threshold computed by the detector is independent of the tested sample. The sample set is simultaneously applied to both a cell-averaging and a ranking & selecting processing techniques. The outputs of these signal processors represent two independent estimates of the unknown noise power level. The two estimates of the noise power are then combined through a general operation to extract the final estimate of the unknown noise level "Z". The resulting value is then multiplied by a predetermined detection coefficient "T" to become a detection threshold against which the content of the tested sample is compared to decide if this content has a target return or it belongs to a pure noise. A sample that exceeds this detection threshold is declared to be detected.

Generally, the CFAR processor is designed to detect signals in the presence of broadband noise and various strong interferers while keeping a constant rate of false alarm. If the noise at the input of the square-law device is a Gaussian narrowband process, each noise sample at the output of the envelope detector is therefore a random variable "x" with an exponential probability density function (PDF):

\[ f_x(x) = \frac{1}{\psi} \exp\left(-\frac{x}{\psi}\right) U(x) \]  \hspace{1cm} (1)

In the above expression, \( \psi \) represents the noise power, and \( U(.) \) denotes the unit-step function.

When Gaussian white noise is modulated onto a carrier and passed through an IF filter, the noise output voltage envelope follows in its statistical behavior to Rayleigh distribution. The signal at the input of the detector, on the other hand, is assumed to be a sine wave with a uniformly distributed phase, over \([0,2\pi]\), and a Rayleigh distributed amplitude S, with a PDF of the form

\[ f_s(s) = \frac{s}{\sqrt{2\pi}} \exp\left(-\frac{s^2}{2}\right) U(s) \]  \hspace{1cm} (2)

It is assumed that the signal has a power of \( S^2 \). This model of fluctuating signal is widely accepted in the radar literature for describing a signal returning from a complex target consisting of many independent scatterers of approximately equal echoing areas. On the other hand, fluctuation of the amplitude of the received signal with a Rayleigh PDF is frequently used as a model for a fading channel.

Conditioned on the value of the amplitude, each sample "x" that originates from a signal at the input of the detector is a random variable with a non-central \( \chi^2 \) PDF [8].

\[ f_x(x) = \frac{1}{\psi} \exp\left(-\frac{x^2}{\psi}\right) I_0\left(\frac{2sx}{\psi}\right) U(x) \] \hspace{1cm} (3)

\( I_0(.) \) stands for the zero-order modified Bessel function of the first kind.

Owing to the simplicity of implementation of the non-coherent integration, it is the most widely type of pulse integration used in radar systems. If the returns of M pulses are now non-coherently integrated, the integrator output takes the form

\[ X = \sum_{i=1}^{M} x_i \] \hspace{1cm} (4)

Each random variable in that sequence has a PDF similar to that given in Eq.(3). The moment generating function (MGF) associated with that PDF has a mathematical form given by:

\[ \Omega_{x}(\omega/A)=\frac{1}{1+\psi\omega} \exp\left(-\frac{\psi A \omega}{1+\psi \omega}\right) \] \hspace{1cm} (5)

In the above expression, \( A=S^2/\psi \) represents the average signal-to-noise ratio (SNR). Since the random variables \( x_i \)'s are assumed to be statistically independent, the integrator output X has a MGF given by...
\[
\Omega_x(\omega/\sigma) = \left(\frac{1}{1 + \psi \omega}\right)^M \exp\left(-\frac{\psi \sigma \omega}{1 + \psi \omega}\right) \quad (6)
\]

The parameter \(\sigma\) is the total, M-pulse, SNR; \(\sigma = MA\) in terms of \(A\), the single pulse SNR. The unconditional MGF can be obtained by averaging Eq.(6) over the target fluctuation distribution of \(\sigma\). For the \(\chi^2\) family of target models proposed by Swerling, the PDF of target fluctuation parameter, and \(\kappa\) represents the degree of signal strength fluctuation. In particular, \(\kappa = 1, 2, 2M\) and \(\infty\) correspond to the Swerling models I, II, III, IV and the non-fluctuating case, respectively. Therefore, the unconditional MGF of the test cell variate, for the \(\chi^2\) family of fluctuating targets, can be easily computed by calculating the average value of Eq.(6) taking into account the PDF of \(\sigma\). Thus,

\[
\Omega_z(\omega) = \left(\frac{1}{\psi}\right)^M \left(\frac{\omega + 1}{\kappa}\right)^{M-1} \exp\left(-\frac{\kappa \omega}{\kappa + \omega}\right) \quad (8)
\]

Now, we turn our attention to the adaptive detection technique. CFAR detection is one of the most desirable features for modern radar receivers, where the variations in the environment characteristics such as the background noise power leads to a severe degradation in the detection performance or an increase in the false alarm rate if a fixed threshold detector is employed in these situations. Fig.(1) depicts the architecture of the underlined CFAR detection procedure. It uses an adaptive threshold whose level is determined by the clutter and/or noise power level, and \(T\) indicates the constant scale factor that must be multiplied by the noise level to construct a detection threshold that guarantees a given rate of false alarm. For fluctuating targets following SWII model, \(v\) has a MGF given by Eq.(8) after replacing \(\kappa\) by \(M\). In other words, the SWII fluctuating target is characterized by:

\[
\Omega_v(\omega) = \left[\frac{\alpha}{\omega + \alpha}\right]^M \quad \text{with} \quad \alpha = \frac{1}{\psi (1 + \gamma)} \quad \text{and} \quad \Delta = \frac{\Lambda}{M} \quad (10)
\]

Based on the previous decision rule, the processor detection performance can be evaluated from the well known relation:

\[
P_d = \int_0^\infty f Z_f (\omega) \int_0^\infty f \frac{Z_f}{T} (\nu) d\nu du \quad (11)
\]

Since \(v\) and \(Z_f\) are statistically independent, letting \(\theta = v\) yields

\[
\Omega_v(\omega) = \Omega_v(\omega) \Omega_{Z_f} (-T \omega) \quad (12)
\]

The substitution of \(\theta\) in the expression of \(P_d\) gives

\[
P_d = \int_0^\infty f \theta (z) dz \quad (13)
\]

\(f_\theta (v)\) represents the PDF of the random variable \(\theta\) which can be computed by performing the Laplace inversion of Eq.(12). Thus, performing this inversion and integrating the resulting form with an allowable change in the order of integration yields:

\[
P_d = \sum \text{res} \left[ \Omega_v(\omega) \frac{\Omega_{Z_f} (-T \omega)}{\omega}, \omega_i \right] \quad (14)
\]

where the contour of integration lies to the right of all singularities of \(\Omega_v(\omega)\) in the left half plane and \(\omega_i\)'s \((i = 1, 2, \ldots)\) are the poles of \(\Omega_v(\omega)\) and \text{res}[\_\_\_] stands for the residue. For the SWII target fluctuation model, the detection probability can be calculated by substituting Eq.(8), after replacing \(\kappa\) with \(M\), in Eq.(14) which yields
probability \( P_d \) occur occasionally in radar signal processing when two research. On the other hand, multiple target situations treatment of this problem is out the scope of the present background with target return, respectively. The from clutter background or from relatively clear degradation in an adaptive threshold scheme leading to excessive false alarms or serious target masking. Consequently masking of one target by the others is called suppression. These interferers can arise from either real object returns or pulsed noise jamming. From a statistical point of view, this implies that the reference samples, although still independent of one another, are no longer identically distributed.

\[
P_d = \left( \frac{1}{\psi} \right)^{\frac{1}{\psi} + \Lambda/M} \lim_{\mu \to \infty} \frac{1}{\mu} \int_0^\infty \frac{d^{\mu-1}}{d\alpha^{\mu-1}} \left( \frac{1}{\alpha} \right) \Psi Z_r (\omega) \bigg|_{\omega = \frac{\psi}{\omega + \Lambda}} - \frac{\psi}{\omega + \Lambda}
\]

(15)

Where \( \Psi Z_r (\omega) \) stands for the Laplace transformation of the cumulative distribution function (CDF) of the noise level estimate \( Z \) and \( \gamma \) represents the average per pulse SNR (\( \gamma = \lambda / M \)).

From Eq.(15), it is evident that the key step in the processor performance evaluation is the determination of the MGF of its noise power level \( Z \) and therefore, we focus our attention, in the following section, on deriving it for the new versions of CFAR detectors when it is operated in multiple-target environments from which the homogeneous performance can be easily obtained as a special case by eliminating the number of extraneous targets. Therefore, it is required now to derive the MGF of the final noise level estimate for each one of the modified versions in order to analytically evaluate its performance either in the absence or in the presence of outliers. Finally, as \( \gamma \) tends to zero (\( \gamma \to 0 \)), Eq.(15) leads to false alarm probability (\( P_d \to P_a \)).

3. MULTITARGET STATISTICAL ANALYSIS OF ADAPTIVE ALGORITHM

The CFAR algorithms were originally developed using a statistical model of uniform background noise. However, this is not representative of real situations. Clutter edges are used to describe transition areas between regions with very different noise characteristics [14]. This situation occurs when the total noise power received within a single reference window changes abruptly. The presence of such a clutter edge may result in severe performance degradation in an adaptive threshold scheme leading to excessive false alarms or serious target masking depending upon whether the cell under test is a sample from clutter background or from relatively clear background with target return, respectively. The treatment of this problem is out the scope of the present research. On the other hand, multiple target situations occur occasionally in radar signal processing when two or more targets are at a very similar range. The consequent masking of one target by the others is called suppression. These interferers can arise from either real object returns or pulsed noise jamming. From a statistical point of view, this implies that the reference samples, although still independent of one another, are no longer identically distributed.

3.1. Cell-Averaging (CA) Algorithm

This adaptive scheme plays an effective part in much noise and clutter environments, and provides nearly the best ability of signal detection while preserving the enough constant rate of false alarm. Generally, in order to analyze the CFAR detection performance when the candidates of the reference window no longer contain radar returns from a homogeneous background, the assumption of statistical independence of the reference cells is retained. The amplitudes of all the targets present amongst the candidates of the reference window are assumed to be of the same strength and to fluctuate in accordance with SWII fluctuation model as the primary target. To start or analysis, let us assume that the reference set contains \( r \) cells from outlying targets with interference level of \( \psi (1+I) \), with \( I \) denotes the interference-to-thermal noise ratio (INR), and \( q = N - r \) cells from clear background with noise power "\( \psi \)". Thus, the total noise power level is estimated as:

\[
Z_{CA} = \frac{1}{N} \left( \sum_{i=1}^{N} X \right) + \frac{1}{N} \left( \sum_{i=r+1}^{N} Y \right) + \Delta Z_h + Z_r
\]

(16)

The random variable \( Z_r \) representing the extraneous target return and that denoting the clear background \( Z_h \) have MGF’s given by a similar form as that indicated in Eq.(10) after minor changing of its parameters. Since the candidates of each type are assumed to be statistically independent, we have

\[
\Omega_{Z_r} (\omega) = \left( \frac{\beta}{\omega + \beta} \right)^{\frac{R}{\omega}} \bigg|_{\omega = \frac{\psi}{\omega + \beta}} \text{ with } \beta = \frac{1}{\psi (1+I)} \text{ & } R = M r
\]

And

\[
\Omega_{Z_h} (\omega) = \left( \frac{\epsilon}{\omega + \epsilon} \right)^{\frac{Q}{\omega}} \bigg|_{\omega = \frac{\psi}{\omega + \epsilon}} \text{ with } \epsilon = \frac{1}{\psi \Delta N} \text{ & } Q = M q
\]

(18)

Since \( Z_r \) and \( Z_h \) are statistically independent, the MGF of \( Z_{CA} \) is simply the product of their individual MGF’s. Therefore, the final noise power level has a MGF of the form:

\[
\Omega_{Z_{CA}} (\omega) = \left( \frac{\xi}{\omega + \xi} \right)^{\frac{R}{\omega}} \left( \frac{\xi}{\omega + \xi} \right)^{\frac{Q}{\omega}} \xi^\frac{\Delta N \beta \& \xi^\frac{\Delta N \epsilon}{2}}
\]

(19)

If the CFAR scheme processes data from M-sweeps, its MGF becomes:

In terms of the MGF of the noise level estimate, the Laplace transformation of the CDF of this estimate can be obtained with the help of Eq.(15) which yields
In order to evaluate the \( k \)-th derivative of Eq.\((20)\), as Eq.\((15)\) demands, the above expression can be put in another mathematical formula using the partial fraction method which leads to:

\[
\Psi_{Z_1}(\omega) = \frac{1}{\omega} \left( \frac{\zeta}{\omega + \zeta} \right)^{R} \left( \frac{\zeta}{\omega + \zeta} \right)^{Q} \left( \frac{\zeta}{\omega + \zeta} \right)^{Q}
\]

(20)

Once the \( k \)-th derivative of the \( \omega \)-domain representation of the CDF of the final noise power level estimate is calculated, the processor performance evaluation can be easily obtained as Eq.\((15)\) demonstrates.

\[\frac{1}{\Gamma(k+i)} \frac{d^k}{d\omega^k} \Psi_{Z_1}(\omega) = \left\{ \begin{array}{ll}
\sum_{i=0}^{\infty} \sum_{j=0}^{\infty} \frac{A_k}{(\omega + \zeta)^{R+k+i+j}} \left( \frac{\zeta}{\omega + \zeta} \right)^{Q-j-k} & \text{for } n = 0 \\
\sum_{i=0}^{\infty} \sum_{j=0}^{\infty} \frac{B_k}{(\omega + \zeta)^{R+k+i+j}} \left( \frac{\zeta}{\omega + \zeta} \right)^{Q-j-k} & \text{for } n > 0
\end{array} \right.\]

(25)

\subsection{3.2. Ordered-Statistics (OS) Algorithm}

The OS technique possesses the good ability to counter the multiple targets. It presents good performance with minor degradation, relative to CA scheme, when the background environment is ideal for much less performance degradation if the operating environment is contaminated with a number of spurious targets provided that their number is within an allowable range. In other words, the OS procedure has its immunity to the presence of outlying target returns amongst the reference cells as long as their density doesn’t exceed the allowable range.

In order to analyze the processor performance in non-homogeneous background, we follow the same steps as those presented for CA technique. Consider the same previously stated situation where there are \( r \) reference samples contaminated by extraneous target returns, each with power level \( \psi(1+i) \), and the remaining \( q=N-r \) reference cells contain thermal noise only with power level \( \psi \). Under these assumptions, the \( K^2 \) ordered sample, which represents the noise power level estimate in the OS detector, has a CDF given by \([9]\):

\[F_{Z_{\psi}}(z) = \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} \frac{q!}{(r-i)!} \left( \frac{1}{\Gamma(M+i)} \right)^{i} \left( \frac{1}{\Gamma(M+j)} \right)^{j} \left( \frac{1}{\Gamma(M+i+j)} \right)^{z} \left( [1 - F_{\psi}(z)]^{i} \right)^{-i} \left( [1 - F_{\psi}(z)]^{j} \right)^{-j} \]

(26)

\( F_{\psi}(z) \) denotes the CDF of the reference cell that contains a homogeneous background and \( F_{\psi}(z) \) denotes the same thing for the reference cell that belongs to interfering target return. Mathematically, these functions can be easily obtained from Eq.\((10)\) after replacing \( q \) by zero and \( 1 \), respectively, and carrying out the Laplace inverse for the resulting formulas. Thus,

\[F_{\psi}(z) = L^{-1} \left\{ \frac{1}{\omega} \left( \frac{\gamma}{\omega + \zeta} \right)^{M} \right\} \quad \text{and} \quad F_{\psi}(z) = L^{-1} \left\{ \frac{1}{\omega} \left( \frac{\beta}{\omega + \zeta} \right)^{M} \right\} \]

(27)

\( L^{-1} \) stands for the Laplace inverse operator. Carrying out the required mathematical operations yields:

\[F_{\psi}(z) = 1 - \sum_{i=0}^{M} \frac{\gamma^i}{\Gamma(M+i)} z^{M+i} \exp(-\gamma z) \]

(28)

\[F_{\psi}(z) = 1 - \sum_{i=0}^{M} \frac{\beta^i}{\Gamma(M+i)} z^{M+i} \exp(-\beta z) \]

(29)

In order to facilitate the Laplace transformation of Eq.\((26)\), we reformulate it as:

\[F_{Z_{\psi}}(z) = \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} \frac{q!}{(r-i)!} \left( \frac{1}{\Gamma(M+i)} \right)^{i} \left( \frac{1}{\Gamma(M+j)} \right)^{j} \left( \frac{1}{\Gamma(M+i+j)} \right)^{z} \left( [1 - F_{\psi}(z)]^{i} \right)^{-i} \left( [1 - F_{\psi}(z)]^{j} \right)^{-j} \]

(30)

The substitution of Eqs.\((28) \& (29)\) into Eq.\((30)\) leads to the result:
The ω-domain representation of the above expression can be obtained by calculating its Laplace transformation which yields:

\[
\Psi_{Z_{OS}}(\omega) = \sum_{i=K}^{N} \max(j, \min(0, r)) \left[ \sum_{j}^{\min(i, q)} \left( r_{j} - j \right) \sum_{m=0}^{i-j} \sum_{n=0}^{j} \left( 1 \right)^{1} \right]
\]

In the above expression, L denotes the Laplace transform operator, whilst * stands for the convolution.

From the mathematical properties of Laplace transformation, it is well-known that [6]

\[
\Theta_{G}(\omega, c, H) \Delta L \left[ \sum_{i=0}^{M-1} \left( \frac{c z}{\Gamma(i+1)} \right) \exp(-c z) \right]^{H} = H
\]

\[
\sum_{\lambda_{0}=0}^{H} \sum_{\lambda_{1}=0}^{H} \sum_{\lambda_{2}=0}^{H} \sum_{\lambda_{M-1}=0}^{H} \Lambda[H; \lambda_{0}, \lambda_{1}, \lambda_{2}, ..., \lambda_{M-1}] \left( \frac{\sum_{j=0}^{M-1} \lambda_{j} + 1}{\prod_{j=0}^{M-1} \lambda_{j}^{\lambda_{j} + 1}} \right) C \sum_{i=0}^{H} \lambda_{i}^{H}
\]

\[
\Omega_{Z_{OS}}(\omega) = \sum_{i=K}^{N} \max(j, \min(0, r)) \left[ \sum_{j}^{\min(i, q)} \left( r_{j} - j \right) \sum_{m=0}^{i-j} \sum_{n=0}^{j} \left( 1 \right)^{1} \right]
\]

Using the identity of Eq. (33), the formulas of (32) can be written as:

Using the identity of Eq. (34), the noise power level estimate of OS scheme takes the form:

\[
\Theta_{G}(\omega, c, H) * \Theta_{G}(\omega, \beta, r + n + j - i)
\]
In order to evaluate the M-sweeps OS performance, we need to calculate the \( \eta \)th derivative with respect to \( \omega \) as Eq.(15) demonstrates. In carrying out this derivative, we obtain

\[
\frac{1}{\eta!} \frac{d^\eta}{d\omega^\eta} \Psi_{Z_{os}}(\omega) = \sum_{i=K}^{N} \sum_{j=\max(0,i-r)}^{\min(i,q)} q^j \sum_{i-j}^{r-j} \sum_{j=0}^{m} j^m \sum_{i-j}^{r-j} \sum_{n=0}^{n} n^n (-1)^{m+n} \\
\sum_{\lambda_0=0}^{q+m-j} \sum_{\lambda_1=0}^{q+m-j} \ldots \sum_{\lambda_{M-1}=0}^{q+m-j} \Lambda(\frac{q+m-j;\lambda_0,\lambda_1,\ldots,\lambda_{M-1}}{\prod_{\ell=0}^{M-1}(\Gamma(\ell+1))^{x^i}}) (\varepsilon)^{\sum_{i=0}^{M-1} x^i} \\
\sum_{\zeta_0=0}^{r+n-j-i} \sum_{\zeta_1=0}^{r+n-j-i} \ldots \sum_{\zeta_{M-1}=0}^{r+n-j-i} \sum_{M-1}^{r+n-j-i} \sum_{\ell=0}^{1} \frac{\Lambda(\frac{r+n-j-i;\zeta_0,\zeta_1,\ldots,\zeta_{M-1}}{\prod_{\ell=0}^{M-1}(\Gamma(\ell+1))^{x^i}}) (\beta)^{\sum_{i=0}^{M-1} x^i}}{\sum_{\mu=0}^{M-1} n^n \sum_{v=0}^{v^n} n^n + v^n + 1} \\
\{\omega + \varepsilon(q+m-j) + \beta(r+n+j-i)\}^{\sum_{i=0}^{M-1} n^n \sum_{v=0}^{v^n} n^n + v^n + 1}
\]

Where

\[
\Phi_{\eta}\left(\xi\right) = \frac{\Delta(\xi)}{\Gamma(\eta+1)}
\]

Where the Pochhammer symbol \((\xi)_\eta\) is as previously defined in Eq.(24)

\[
(\xi)^\Delta = \frac{(\xi + \eta - 1)!}{(\xi - 1)!} = \eta! \left(\frac{\xi + \eta - 1}{\xi - 1}\right)
\]

Since the \((M-1)^{th}\) derivative of the Laplace transformation of the CDF of the noise level estimation is the heart of the processor performance evaluation, the OS behavior against the detection of \(\chi^2\)-fluctuating target is completely analyzed, as Eq.(15) demonstrates, where the false alarm and detection probabilities are dependent on this derivative.

### 3.3. Modified Versions

The CA scheme is the optimum CFAR processor which gives the highest detection performance in the case where the estimation cells contain independent and identically distributed observations governed by an exponential distribution, given that the operating...
environment is free of any interferers and the background noise is homogeneous. On the other hand, the OS scheme has its immunity to the presence of extraneous target returns amongst the contents of the noise level estimation cells as long as their density of existence is within an allowable range. Therefore, it is intuitive to use these identities of CA and OS schemes to develop new versions of adaptive algorithms. In these developed versions, it is of importance to overcome the drawbacks offered by the CA algorithm in heterogeneity where excessive numbers of false alarms at clutter edges and severe performance degradation in the presence of spurious targets. Additionally, it is required to improve the homogeneous detection performance of the OS technique in order to obtain a processor that acts as the CA algorithm in ideal situation and behaves as the OS scheme in heterogeneity situation caused either by clutter edges or multitargets. Here, we are interesting in analyzing three of such derived versions; namely: Average-Of (AO), Greatest-Of (GO) and Smallest-Of (SO) processors. In these techniques, there are two independent estimators of the background noise level; one uses the CA rule and the other employs the OS procedure. The resulting two noise level estimators are then compared to obtain the average, the maximum, or the minimum one of the two values to represent the final estimation of the unknown noise power level.

a) Average-Of (AO) Algorithm

The contents of the reference window of size N samples feed two individual signal processors. The first one of these processors estimates the unknown noise power level employing the CA basis while the other does the same thing using the OS rule. The two estimates are combined through the average operation to arrive at the final noise power level estimation. Thus,

\[
Z_{AO} \triangleq \text{Average}(Z_{CA}, Z_{OS})
\]

(40)

The \(\omega\)-domain representation of \(Z_{AO}\) is characterized by a MGF of the form:

\[
\Omega_{Z_{AO}}(\omega) = \frac{1}{2} \left( \Omega_{Z_{CA}}(\omega) + \Omega_{Z_{OS}}(\omega) \right)
\]

(41)

All the parameters of the above equation as well as their \((M-1)^{th}\) derivatives are previously evaluated. Therefore, the processor detection performance is completely determined as illustrated in Eq.(15).

b) Greatest-Of (GO) Algorithm

This version was specifically aimed at reducing the number of excessive false alarms at clutter edges. For this algorithm, when the cell under test is greater than both of the two CA-CFAR and OS-CFAR thresholds, a target present will be declared. Otherwise, no target will be declared. This is equivalent to choosing the maximum value of the CA- and the OS-CFAR thresholds and compare it with the target cell (cell under investigation) to indicate whether or not the target is present. In other words, the final noise power is estimated from the larger of the two separate noise level estimates: one of which uses the rule of CA technique while the other employs the theory of OS procedure. The maximum one of these two estimates is selected to represent the final noise power level. This means that \(Z_t\) is obtained through the relation:

\[
Z_{GO} \triangleq \text{GO}(Z_{CA}, Z_{OS})
\]

(42)

In this case, \(Z_t\) has a CDF given by

\[
F_{Z_{GO}}(z) = F_{Z_{CA}}(z) F_{Z_{OS}}(z)
\]

(43)

In order to analyze this version, the Laplace transformation of the above formula must be computed. To carry out this task, we start with calculating the Laplace inverse of Eq.(21) which yields

\[
F_{Z_{CA}}(z) = 1 - \sum_{k=0}^{R-1} \frac{1}{\Gamma(R-k)} \left(-z\right)^{R-k-1} \exp\left(-\xi z\right)
\]

(44)

By taking the Laplace transformation of Eq.(45), one obtains:

\[
F_{Z_{GO}}(z) = F_{Z_{CA}}(z) \cdot F_{Z_{OS}}(z)
\]

(45)

By taking the Laplace transformation of Eq.(45), one obtains:

\[
\psi_{Z_{GO}}(\omega) \triangleq \mathbb{L}\{F_{Z_{GO}}(z)\} = \psi_{Z_{CA}}(\omega) \cdot \psi_{Z_{OS}}(\omega)
\]

(46)

The \((M-1)^{th}\) derivative of the above expression is a straightforward. Finally, the substitution of this derivative into the definition of \(P_0\) yields the evaluation of the processor performance of the underlined CFAR algorithm.

c) Smallest-Of (SO) Algorithm

In order to prevent the suppression of closely spaced targets, the minimum version has been introduced. While testing for target presence at a particular range, the processor must not be influenced by the outlying target echoes. For this procedure, a target present will
be declared if the cell under test is greater than any of the two thresholds. Otherwise a no target declaration will be indicated. As the case of SO-CFAR, the content of the target cell should be greater than the CA- or the OS-CFAR threshold to declare a target present. This is equivalent to selecting the minimum value of the CA- and the OS-CFAR thresholds and compares it with the primary target return to know its presence or absence. This means that the noise power estimate is achieved by taking the smaller one of the two estimates obtained through CA and OS basics as depicted in [13]. That is,

\[ Z_{SO} \triangleq \min \{ Z_{CA}, Z_{OS} \} \]  

(47)

In this case, the final noise level estimate has a CDF given by [7]

\[ F_{Z_{SO}}(z) = F_{Z_{CA}}(z) + F_{Z_{OS}}(z) - F_{Z_{CA}}(z) - F_{Z_{OS}}(z) \]  

(48)

It is obvious from the above formula that there is a direct relation between the detection performance of GO- and that of SO-CFAR algorithms. This means that once the performance of GO version is evaluated, the performance of SO procedure is easily obtained. In o- domain, Eq.(48) takes the form

\[ \Psi_{Z_{SO}}(\omega) = \Psi_{Z_{CA}}(\omega) + \Psi_{Z_{OS}}(\omega) - \Psi_{Z_{CA}}(\omega) - \Psi_{Z_{OS}}(\omega) \]  

(49)

Again, once the Laplace transformation of the CDF of the noise level estimate is achieved, the false alarm rate performance as well as the detection probability in the absence, or the presence, of spurious targets are completely determined since Eq.(49) represents the backbone of their evaluation.

4. STATISTICAL RESULTS OF ADAPTIVE ALGORITHM

Here, we provide a variety of detection characteristics for the CA- and OS-CFAR processors as well as their derived versions in the case where the operating environment contains homogeneous background or contaminated with a number of extraneous targets along with the target under test. In our presented results, we take the behavior of the CA scheme as a reference performance against which the performance of any one of the underlined processors is compared to see to what extent the developed version can improve the homogeneous performance of the CA algorithm. On the other hand, the performance of the OS technique is taken as a reference one against which the multitarget performance of the modified version is compared to see which version has improved reaction against the presence of outlying target returns amongst the candidates of the reference set. For a given size of the reference set, the performance of the OS scheme depends mainly on the rank of the selected cell whilst all the elements of the reference set are combined to estimate the unknown noise power level in the case of CA processor. Therefore, it is of importance to have an OS detector with highest performance. For a reference window of size 24, the optimum value of K that gives top performance is 21 [4]. The design probability of false alarm is maintained constant at 10^-6. Fig.(2) depicts the thresholding constant as a function of the ranking order parameter "K" when there are two and four integrated pulses. From the displayed results, it is shown that the CA scheme has an unchanged T irrespective to K while the OS technique has a varied T which decreases as K increases. The ML version gives slow varying T factor below that of the CA detector, the MX operator behaves like CA for lower values of K and decreases slowly as K increases and tends to act as the OS algorithm for higher values of K. On the other hand, the MN version has scale factor T equals to that of OS detector for lower values of K and decreases rapidly as K increases and tends to behave as CA scheme near the upper limit of the ranking order parameter. This behavior of T for the GO operator is predicted since this version selects the maximum of the two noise level estimates which in turn results in choosing the minimum of their corresponding T values for the false alarm rate to be held constant. On the other hand, since the SO version chooses the minimum one of the CA and OS noise level estimates, its constant scale factor must be the larger one of their T values to maintain the same rate of false alarm. For lower values of K, the averaging operator gives an estimated noise power level which is approximately equal to that of the CA and, because of this, its T values lie near those of the CA. As K increases, the estimated noise power increases accordingly and this leads to decrease the thresholding constant in order to keep the false alarm rate unchanged. The variation of this family of curves illustrates the utility of non-coherence integration of M pulses where lower threshold values and consequently higher detection performances are obtained by increasing the number of non-coherently integrated pulses. This property is common for any CFAR detection scheme.

Let us now turn our attention to the detection characteristics of the examined CFAR processors. From this point of view, our numerical results are partitioned into two categories. In the first group of figures, the background environment is assumed to be ideal; i.e. it is free of any interferers except the background noise and clutter. The family of this category includes Figs. (3-6). These plots illustrate the detection performance of the derived versions as well as the OS(21) scheme for a number of integrated pulses of 2, 3, 4, and 5. To see to what extent the non-coherent integration can improve the adaptive detection
performance, we incorporate the single sweep (M=1) performance into the family of curves of each figure. Since the CA processor is the optimum one in the world of CFAR techniques that gives highest detection performance [13] when the operating environment is homogeneous, we include its performance, under the same circumstances, with that of any other version as a reference for comparison. Fig. (3) shows the detection probability of GO-CFAR algorithm as a function of the primary target’s SNR for M sweeps when the tested target fluctuates obeying in its fluctuation SWII model. The optimum value of K for this version of the derived algorithms is 15 [12]. Therefore, the underlined figure depicts the performance of MX(15) algorithm along with that of CA scheme to see to what extent the behavior of this version can exceed or may surpass the reaction of the optimum processor in the CFAR world when operates in ideal background conditions. The inspection of the family of curves of this plot shows that the performance of the algorithm under investigation is higher than that of the CA detector which is the king of adaptive techniques. This indicates that, by combining the OS and CA processors, we can attain a detection performance which may not only exceed but also may surpass that of CA procedure. Additionally, the rate of improvement decreases as the number of integrated pulses increases. Fig. (4) illustrates the same thing for the SO-CFAR processor in the case where the ordered sample has an optimum rank of 18 [12]. There is a minor improvement, relative to the performance of CA algorithm, of the performance of MN(18). As the number of consecutive sweeps augments, the reaction of MN(18) tends to be the same as the CA scheme. Fig.(5) displays the same numerical results for the averaging operator. The ordered sample, that has the highest detection performance, has a rank of 17 as [12] demonstrates. The family of curves of this plot shows that there is a noticeable improvement than in the case of MX(15) or MN(18). This means that the AO operator gives the highest, relative to the GO or the SO operator, improvement. In addition, the obtained improvement is evident in the case of averaging operator than in the other two cases; especially when the number of non-coherently integrated pulses increases. Finally, in order to show to what extent the derived versions improve the detection performance of the well-known adaptive techniques, the last figure in this category depicts the reaction of the standard OS algorithm against the homogeneous background in which the searching target is located. As predicted, there is a noticeable degradation in the OS performance and this degradation continues even with non-coherent integration of M pulses as Fig.(6) demonstrates. From the displayed results we arrive at the conclusion that: by comparing the noise level estimates through CA and OS schemes, we can obtain an adaptive algorithm the homogeneous detection performance of which can surpass the optimum adaptive technique which is the standard CA procedure. The displayed results in Fig.(7) confirm this conclusion. It depicts the required SNR, to achieve an operating point of (P_{false}=90\%, \ P_{true}=10^{-6}) attained by the CFAR algorithms when the radar receiver non-coherently integrates M pulses. The optimum processor is incorporated among the curves of this plot to see which version is capable to react like the fixed-threshold processor in detecting fluctuating targets. The presence of multiple targets represents the critical situation in evaluating the performance of adaptive detectors. In order to take the OS scheme as a reference against which the performance of the derived versions is compared, it is of importance to research the highest multitarget detection performance of such technique for measuring the behavior of the derived versions to see if they behave below, like, or surpass this standard adaptive technique. Since the OS algorithm presents the same immunity to outlying targets irrespective to their number as long as this number is less than or equal to the difference between the size of the reference set 'N' and the ranking order parameter 'K', we are going to evaluate the multiple-target detection performance of the maximum allowable value for the interfering target returns which is 3 (r ≤ N-K). The detection performance of the CA and OS as well as their derived versions is evaluated for the case where the operating environment is contaminated with three outliers and the numerical results are displayed in the second category of curves. This family of figures includes Figs.(8-11). Fig.(8) distinctly illustrates the detection performance of the GO-CFAR scheme for this situation of operating conditions. The displayed curves are obtained for a possible practical application of equal strengths for the outlying as well as the primary target (INR=SNR). From the variations of the underlined curves, it is obvious that the MAX algorithm reacts worst when it operates in multiple-target environment. Additionally, there is an improvement in the processor performance as the number of integrated pulses increases, as predicted. Moreover, the rate of improvement decreases as the number of consecutive sweeps augments. Furthermore, this improvement tends to be negligible as M becomes large. Fig.(9) depicts the same thing for the SO version under the aforementioned assumption. By inspecting the family of curves of this plot, it is evident that the reaction of this algorithm surpasses that of the OS scheme under the same operating conditions. The average AO operator, on the other hand, gives a multiple-target performance higher than that of GO
version and less than that of SO procedure, as Fig.(10) demonstrates. Finally, it is of interesting to trace the behavior of the CA scheme against the presence of extraneous target returns amongst the estimation cells. Fig.(11) displays the multitarget detection performance of the well-known CA processor for the same parameter values as the other CFAR algorithms. The underlined plot distinctly illustrates how can the CA technique degrades the multiple-target detection performance whereas it gives optimum adaptive performance in homogeneous situation. As well-known, the non-coherent integration of M pulses, improves the detection performance of the adaptive processor either it operates in homogeneous or multi-target environment, as the candidates of the two categories of figures demonstrate. The AO and GO versions have lower values for the detection probability than the OS procedure. Additionally, the SO algorithm is the only one that has a multiple-target detection performance higher than that of the OS scheme. These concluded remarks are associated with the steady-state behavior of the underlined processors.

Since each hybrid version has a specified value for its optimum ranking parameter K, it is preferable to evaluate its reaction against interfering targets for that value as well as the optimum ranking value for the original OS scheme. This is actually what we are going to do it in the displayed results of Fig.(12). The examination of this family of curves shows that AO and GO algorithms present better behavior against extraneous targets for K=21 than their optimum values 17 and 15, respectively, whilst the SO procedure has a revere action. Moreover, the performance of SO(18) surpasses the performance of its mother scheme OS(21). As predicted, the CA scheme gives the worst detection performance. Using the same representation, we denote the CA processor by CA(24) which means that the cell averaging of 24 reference samples is taken as an estimate of the unknown noise power level. It is of importance to note that the family of curves of the current plot is obtained through the integration, non-coherently, of two pulses (M=2). Additionally, the CA technique offers, approximately, the same reaction to the interferers as the GO(15) algorithm, but still less than GO(21).

Finally, the variations of the false alarm rate with the strength of the interfering target returns are plotted for the CA and OS architectures long with their hybrid versions in the presence of non-coherent integration of two consecutive pulses (M=2). In this situation, it is assumed that there are three samples contaminated with spurious target returns amongst the elements of the reference set of size 24 and the design rate of false alarm is held constant at its nominal value ($10^{-6}$). The variations of the curves of this figure show that the SO(18) hybrid algorithm is the only one among the derived versions that is capable of maintaining the rate of false alarm constant in multiple-target situations, especially for strong interference level as its mother OS(21) scheme. Also the SO(21) procedure tends to hold its false alarm rate constant but at a more lower level than SO(18). All the other CFAR techniques have degraded false arm rate performance. Additionally, the non-coherent integration makes the false alarm rate performance of CA(24), AO(17), AO(21), GO(15), and GO(21) worst while doesn't affect OS(21), SO(18), and SO(21). This concluded remark is predicted since the outlying targets participate in the threshold construction in first family whilst it has no effect in the second category of CFAR schemes. In other words, the presence of interferers among the estimation cells of the first family pushes the detection threshold towards its higher values and this in turn lowers the false alarm probability. On the other hand, as the number of non-coherently integrated pulses increases, the estimated power of interference increases and consequently the detection threshold raises more and more leading to lowering the false alarm rate more and more. For the candidates of the second family, these interferers have no effect on their false alarm rate since their number rests within its allowable range. Moreover, the OS and SO false alarm rate performances improve as the number of post-detection integrated pulses increases, which is not offered by the CA technique. This result is expected since the largest extraneous target returns occupy the top ranked cells and therefore they are not incorporated in the estimation of the background noise power level. In other words, the noise estimate is free of outlying target returns and therefore it represents the homogeneous background environment. Consequently, the false alarm rate performance of the OS procedure improves as the number of non-coherently integrated pulses increases.

5. CONCLUSION

This paper detailedly evaluate the detection performance of the well-known OS scheme as well as the conventional CA processor along with their new derived versions when the radar receiver non-coherently integrates M consecutive sweeps to decide whether the searching target is present or absent. The background environment, from which these adaptive algorithms obtain their processing data to construct the detection threshold which is the backbone of the processor's decision, is assumed to be free of any interferers or contaminated with some spurious targets. The primary as well as the secondary interfering targets is assumed to be fluctuating in accordance with $\chi^2$-distribution with M degrees of freedom. Three versions of such techniques are processed and closed form
expressions are derived for their detection performance. These procedures include AO, GO, and SO operations on two separately noise power estimates from a reference set of N cells: one of them employs CA technique and the other uses OS basis. As expected, the detection performance of the hybrid versions outweighs that of CA scheme, either in homogeneous or in multi target environments, for some selected values for the ranking order parameter. From the interference point of view, the considered detectors are partitioned into two families: the CA family and the OS one. The family of CA incorporates AO and GO while that of OS includes SO only. The performance of OS family outweighs that of CA family in non-homogeneous situations. In addition, this family is capable of maintaining a constant rate of false alarm, irrespective to the interference level, in the case where the spurious target returns occupy the top ranked cells and they are within their allowable range. As a final conclusion, the detection performance of the modified versions is related to the ranking order parameter, the target model, the average power of the target, and the environmental operating conditions.

The post detection integration has demonstrated its validity as a tool to improve the processor detection performance. Additionally, the rate of improvement decreases as the number of integrated pulses increases and tends to be negligible when this number becomes large. The results and the mathematical treatment may provide some propositional advices for exploring other mathematical rules in developing further versions of adaptive schemes.

REFERENCES

Fig. 1. Block diagram of the new versions of CFAR processors with noncoherent integration

Fig. 2. M-sweeps thresholding constant as a function of the ranking order parameter K for the new version of CFAR detectors when N=24, and Pfa=1.0E-6

Fig. 3. M-sweeps homogeneous detection performance of MAX processor for N=24, and Pfa=1.0E-6

Fig. 4. M-sweeps homogeneous detection performance of MIN processor for N=24, and Pfa=1.0E-6

Fig. 5. M-sweeps homogeneous detection performance of ML processor for N=24, and Pfa=1.0E-6

Fig. 6. M-sweeps homogeneous detection performance of OS processor for N=24, and Pfa=1.0E-6

Fig. 7. M-sweeps required SNR, in homogeneous situation, of the well-known CFAR schemes along with
their developed versions for \( N=24, P_d=0.90, \) and \( P_{fa}=10^{-6} \)

Fig. 8. M-sweeps multitarget detection performance of MAX scheme for \( N=24, r=3 \) and \( P_{fa}=1.0E-6 \)

Fig. 9. M-sweeps multitarget detection performance of MIN scheme for \( N=24, r=3 \) and \( P_{fa}=1.0E-6 \)

Fig. 10. M-sweeps multitarget detection performance of ML scheme for \( N=24, r=3 \) and \( P_{fa}=1.0E-6 \)

Fig. 11. M-sweeps multitarget detection performance of CA processor for \( N=24, r=3 \) and \( P_{fa}=1.0E-6 \)

Fig. 12. M-sweeps multitarget detection performance of adaptive schemes for \( N=24, M=2, r=3, \) INR=SNR, and \( P_{fa}=10^{-6} \)

Fig. 13. Actual probability of false alarm, in multitarget environment, of CFAR processors when \( N=24, M=2, r=3 \), and design \( P_{fa}=10^{-6} \)