Dynamic Modeling of the Centrifugal Compressor for Cathode Entrance of PEM Fuel Cell According to Requested Electrical Current

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ABSTRACT:
Air management of the polymer fuel cell system is a key issue to increase the performance and the efficiency. The air compressor is used as an important lateral device for the fuel cell system. By using of this system, input parameters into the fuel cell collection are determined and used in other subsystems. The final goal of this paper is to design the dynamic model for an air compressor of the fuel cell collection in the Matlab/Simulink software. This action is for obtaining an accurate and continuous control on the input air parameters. In this model, by using of the static feed forward function, requested electrical current of the fuel cell is related to the air compressor model. In addition, input thermodynamic parameters of the air compressor have been assumed to set in the determined value. With the designed model, changes in the compressor parameters are intercepted. Investigated parameters are angular velocity, temperature, compressor power, mass flow rate, outlet air pressure and the compressor efficiency. Finally, the results of the dynamic model as well as the compressor map are fully validated based on the available results in the open literature. Therefore, the centrifugal compressor model is produced that can be added to polymer fuel cell collection as a lateral system. In addition, the results show that the model can predict the dynamic behavior of the compressor accurately and it can be used directly for any control purposes.

KEYWORDS: Compressor, Dynamic process, Pressure ratio, Mass flow rate.

1. INTRODUCTION
According to using fuel cells in the wide ranges, various aspects for using them are rapidly developing. Among the major issues, application of these systems in transportations (automobiles, bus, train, ship etc.), distribution production (supplying electrical power of houses, hotels, restaurants etc.) and mobile devices (cell phone, laptop etc.) have been noted [1]. In this energy sources (PEM fuel cell), the inlet oxidant and fuel to the fuel cell systems are oxygen and hydrogen respectively. Polymer fuel cell system are includes as fuel cell stack (as a heart of system) and the lateral collections. In the lateral collections, various processes are done upon the effective parameters of the fuel cell. Some of these processes include controlling the stack temperature, transfer of the reactant material, supplying the operational pressure of anode and cathode sides etc. This auxiliary equipment is consumer of electrical power in the collection and parasitic effect of them has been considered upon the final power of stack. Air compressor is one of these systems. This device is used for input air of the PEM fuel cell compressor. This unit is exerting the greatest effect on the output voltage of the fuel cell. Various paths for compressing input air of cathode side are includes the use of blower, fan and compressor. Blower and fan are delivering the large amount of mass flow rate of air in the low-pressure ratio. Compressors are more effective to supplying air of the fuel cell. Reason for this issue is the high-pressure ratio (than blowers) in this kind of devices.
Hence, with the use of centrifugal compressor, obtained electrical power density in the fuel cell systems are increased. Therefore, centrifugal compressors have been used in the applications of PEM fuel cell in distributed productions and transportation [2]. This issue demonstrates the importance of the modeling of compressor as the main part of fuel cell subsystem. Several Studies and researches have been done upon the modeling and simulation of the compressor behavior. Pukrushpan et al [3] studied a model, including air submitter compressor for fuel cell cathode input. In this model (according to requested electrical current of the fuel cell), controlling of the compressor has been done accurately. The main reason of this control is for effective supplying of the cathode reactant material. Eghbal et al have done a similar work [4]. In this study, a dynamic model of compressor has been evaluated in the hybrid fuel cell system. In this system, air has been deliver ed to fuel cell system (according to requested electrical current and embedding a novel model of compressor). Furthermore, Tirnovan et al are designed an alternative model for fuel cell compressor. In this model and with the aid of MSL method, alternative compressor model has been presented. The results of this method have been used for developing the fuel cell compressor dynamic model [5].

Moraal et al used the results of dynamic model that was provided in the Jensen & Kristensen [6]. They prepared the comprehensive model of air compressor behavior with the changes in effective parameters. Air compressor parameters are change in angular velocity, power consumption, temperature and the compressor characteristic curve.

The final goal of this paper is to design the dynamic model for an air compressor of the fuel cell collection in the Matlab/Simulink software. Therefore, dynamic and comprehensive model of the air compressor has been presented in this paper. This model has the possibility to adding to the fuel cell system model. Further more, tracking possibility of changes in the dynamic parameters of compressor is such as major benefits of produced model.

2. MATHEMATIC MODEL AND SIMULATION OF THE COMPRESSOR

In the dynamic model, tracking possibility of changes in thermodynamic characteristic of the compressor according to look-up tables is not feasible. Existence of such an issue has led to design dynamic and comprehensive model for investigating changes in air compressor parameters. In this simulation, all effective parameters of the compressor with their equations have been located in the main model.

Fig. 1 shows a schematic model of the compressor. This diagram is for investigating the arrangement, influence and reception of the extracted parameters and equations of the model. Inputs for this model are input pressure $p_{cp,in}$, input air temperature $T_{cp,in}$, output air pressure of compressor $p_{cp,out}$ and compressor voltage. Temperature and pressure of incoming air to compressor are respectively considered one atmosphere and 25°C.

As shown in Fig. 1, the main model is composed of two parts. The first part is the compressor static map that shows the amount of mass flow rate through the compressor. In addition to this parameter, temperature and power of the compressor have been determined in this model. These amounts are determined according to applied thermodynamic equations.

In the second part, compressor angular velocity is calculated according to inertia of motor and compressor. The importance of the compressor angular velocity is the point that should be considered accurately. This parameter is the only dynamic parameter that is used for calculating mass flow rate and the amount of consumed electrical current of the compressor. In this model, the static feed forward function is used to calculating compressor voltage according to requested electrical current of the fuel cell. Hence, based on (1), the compressor voltage is calculated.

$$v_{cm} = 0.67 \times I + 33.55 \quad (1)$$

In the dynamic model, the above equation is used to calculate required torque for driving the compressor. In the above equation, $I$ is the required electrical current of the fuel cell. Whereas changes in the thermodynamic characteristic of compressor does not possible for dynamic simulation (according to look-up tables), dynamic model of compressor is presented according to governed equations and parameters of the compressor [6]. For modeling the compressor characteristics, curve-fitting method has been used in the nonlinear state.
In this study, for modeling the compressor in the dynamic state, Jensen & Kristensen model has been used [6]. To illustrate the changes in the compressor parameters, modified amounts of the mass flow rate and velocity are used for the compressor map. Hence below equations are presented [7].

\[ N_{cr} = N_{cp} / \sqrt{\delta} \]  
\[ W_{cr} = W_{cp} / \sqrt{\delta} \]  
\[ \theta = T_{cp, in} / 288 \]  
\[ \delta = p_{cp, in} / 1 \text{ atm} \]

In the above equation \( N_{cr}, W_{cr}, \theta \) are respectively defined as modified parameters of velocity, mass flow rate, temperature and pressure.

According to Jensen & Kristensen model, non-dimensional parameter of head \( \psi \) is obtained.

\[ C_f T_{cp, in} \left[ \frac{p_{cp, out}}{p_{cp, in}} \right]^{\gamma-1} \left[ \frac{\gamma}{\gamma-1} \right]^{\frac{1}{\gamma-1}} \]

The numerator of the above equation is the electrical power of the compressor static model. For calculating of the head parameter, blades-tip-speed should be calculated. According to modified angular velocity, Blades-tip-speed is obtained from (7).

\[ U_C = \frac{\pi}{60} \frac{d_C}{N_{cr}} \]

Modified mass flow rate is defined as follow [7].

\[ \Phi = \frac{W_{cr}}{\rho_a \frac{\pi}{4} d_C^2 U_C} \]

Modified mass flow rate \( \Phi \) has been related to head parameter \( \psi \) as follow [7].

\[ \Phi = \Phi_{\text{max}} \left( 1 - \exp\left( \frac{\beta}{\psi_{\text{max}}} \right) \right) \]

Which \( \Phi_{\text{max}}, \beta, \psi_{\text{max}} \) are related to Mach number [7] (as a polynomial functions).

\[ \Phi_{\text{max}} = a_4 M^4 + a_3 M^3 + a_2 M^2 + a_1 M + a_0 \]
\[ \beta = b_2 M^2 + b_1 M + b_0 \]
\[ \psi_{\text{max}} = c_5 M^5 + c_4 M^4 + c_3 M^3 + c_2 M^2 + c_1 M + c_0 \]

In addition, Mach number has been defined as follow.

\[ M = \frac{U_C}{\sqrt{\gamma R_a T_{cp, in}}} \]

In the Table 1, the parameters of the compressor diameters, universal gas constant and air density has been presented.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>value</th>
<th>Parameter</th>
<th>value</th>
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</thead>
<tbody>
<tr>
<td>( R_a )</td>
<td>2.869 \times 10^2 \</td>
<td>( \rho_a )</td>
<td>1.23 \ kg/m^3</td>
</tr>
<tr>
<td>( d_c )</td>
<td>0.2286 \ m</td>
<td></td>
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</table>

Coefficients of \( a_i, b_i, c_i \), are the returned coefficients that is presented in Table 2. These coefficients are obtained according to curve fitting method in [7].

<table>
<thead>
<tr>
<th>Parameter</th>
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<th>Parameter</th>
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<tbody>
<tr>
<td>( a_4 )</td>
<td>-3.69906 \times 10^{-5} \</td>
<td>( b_0 )</td>
<td>2.44419</td>
</tr>
<tr>
<td>( a_3 )</td>
<td>2.70399 \times 10^{-4} \</td>
<td>( c_4 )</td>
<td>-9.78755 \times 10^{-3}</td>
</tr>
<tr>
<td>( a_2 )</td>
<td>-5.36235 \times 10^{-4} \</td>
<td>( c_5 )</td>
<td>-0.42937</td>
</tr>
<tr>
<td>( a_1 )</td>
<td>-4.63685 \times 10^{-5} \</td>
<td>( c_2 )</td>
<td>0.10581</td>
</tr>
<tr>
<td>( a_0 )</td>
<td>2.21195 \times 10^{-3} \</td>
<td>( c_3 )</td>
<td>0.80121</td>
</tr>
<tr>
<td>( b_2 )</td>
<td>1.76567 \</td>
<td>( c_1 )</td>
<td>-0.68344</td>
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</tbody>
</table>

Hence, for obtaining the modified mass flow rate, Simulink model of the static part of the compressor has been shown in Fig. 2. In this model, modified mass flow rate with non-dimensional equations has been calculated (equation ((5) and (8)-(13)).

To obtaining the compressor efficiency, (in the Simulink model) look-up tables have been used [7]. According to these tables, the compressor efficiency is related to the pressure ratio and the amount of mass flow rate. Maximum amount of compressor efficiency according to these parameters is 80%. Diagram of this table has been presented in Fig. 3.
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Whereof, temperature inside the compressor increases strongly, changes in the temperature should be tracked for compressor outlet air. Hence, in the (14) change in the temperature is presented in terms of input air temperature and input and output pressure of the compressor [9].

\[
T_{cp, out} = T_{cp, in} + \frac{T_{cp, in}}{\eta_{cp}} \left( \frac{P_{cp, out}}{P_{cp, in}} \right)^{\frac{\zeta-1}{\gamma}} - 1
\]  

(14)

Fig. 3: Changes in the compressor efficiency according to the mass flow rate and pressure ratio

Outlet air pressure of the compressor has been calculated with gas law. According to thermodynamic equations, the amount of the requested torque is calculated for the compressor.

\[
\tau_{cp} = \frac{C_p T_{atm}}{\omega_{cp} \eta_{cp}} \left( \frac{P_{cp, out}}{P_{atm}} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right] W_{cp}
\]

(15)

In the above equation, \(\tau_{cp}\) is the requested torque in terms of N.m, \(C_p = 1004 J/kg^\gamma\) is the specific heat capacity of air and \(\gamma = 1.4\) is the specific heat capacity ratio. Equation (16) has been used to illustrate the dynamic mode of compressor velocity [9].

\[
\frac{d\omega_{cp}}{dt} = (\omega_{cm} - \tau_{cp} \omega_{cp}) \omega_{cp}
\]

(16)

In the above equation (inertia function), \(J_{cp} = 5 \times 10^{-5} kg.m^2\) is the mixed compressor and motor inertia, \(\omega_{cp}\) is the angular velocity rad/sec, \(\tau_{cm}\) is the input motor-compressor torque and \(\tau_{cp}\) is the requested torque N.m that is used to driving the compressor. Requested torque has been obtained according to static motor equation as follow [5].

\[
\tau_{cm} = \eta_{cm} \frac{k_i}{R_{cm}} (v_{cm} - k_v \omega_{cp})
\]

(17)

\(k_v\) are motor \(R_{cm}\) constants and \(\eta_{cm}\) is the mechanical motor efficiency. These coefficients have been presented in Table 3.

<table>
<thead>
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<tr>
<td>(k_v)</td>
<td>0.0153 V/(rad/s)</td>
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<tr>
<td>(k_i)</td>
<td>0.0225 N.m/Amp</td>
</tr>
<tr>
<td>(R_{cm})</td>
<td>0.82Ω</td>
</tr>
<tr>
<td>(\eta_{cm})</td>
<td>7/98</td>
</tr>
</tbody>
</table>

Table 3: compressor motor parameters

The amount of electrical power in the compressor motor model has been calculated according to the following equation.

\[
P_{cm} = \tau_{cm} \times \omega_{cp}
\]

(18)

Hence, due to the extracted equations and the requested electrical current from fuel cell (which is in the relation with the output air mass flow rate of the compressor), changes in the compressor parameters has been obtained.

In this model, with the help of static feed forward function and using the Jensen & Kristensen model, changes in the mass flow rate of air, operational pressure, temperature, electrical power in the static section, electrical power in the dynamic mode, angular velocity and compressor efficiency have been evaluated.

3. RESULTS AND DISCUSSIONS

During this paper, dynamic model of the air compressor was designed according to performed simulation in Matlab/Simulink software. In this simulation, changes in the thermodynamic and electrical parameters of compressor (in dynamic mode) have been originated from an arbitrary electrical current of the fuel cell that is applied to compressor model input. This model is designed in the way that impact and reception of all parameters is visible on each other. Angular velocity (as a dynamic parameter) is the main factor for calculating mass flow rate, electrical power and non-dimensional parameters.

Calculating the angular velocity of the compressor has been done according to difference in the electrical power of the static and dynamic compressor models. Therefore, instantaneous changes of each parameter, shows its effects over other parameters. For instance, enhancement in the electrical current is caused to enhance angular velocity and hereupon consumed electrical current is increased in the compressor. Additionally, enhancement of the electrical power in the inertia function of the motor and compressor model will led to increase the angular velocity. As shown in the above, requested electrical current of the fuel cell,
is the disturbance and input parameter for compressor model. Requested electrical current to the fuel cell system (in the time unit), has been shown in Fig. 4.

![Fig. 4: Time distribution of the fuel cell requested electrical current](image)

Proportional to changes in the electrical current, electrical power of static and dynamic models has been obtained. With the aid of non-dimensional numbers and presented equations, electrical power of the static model has been calculated (according to compressor angular velocity in dynamic mode). Electrical power of the compressor motor model has been obtained with the aid of static feed forward function. Therefore, Fig. 5 shows the electrical power changes for static and dynamic models.

![Fig. 5: changes in the electrical power of compressor-motor and static models](image)

Different amounts in the electrical power, (in the function of compressor inertia) are cause to change in the angular velocity. Therefore, changes in the angular velocity of the compressor have been shown in Fig. 6. As it is obvious, when electrical current is changing, difference in the electrical power is happening. Therefore, according to (16) angular velocity of the compressor (in this time) increases or decreases.

![Fig. 6: changes in compressor angular velocity by using of motor and compressor function](image)

Fig. 7 shows the sample of changes in the parameters of compressor curve. In this curve, bevel lines indicate that angular velocity of compressor is constant. Final goal of modeling the centrifugal compressor is to obtain the curve characteristic. Therefore, non-dimensional numbers have been used for calculating the parameters as angular velocity, mass flow rate and pressure ratio. In this curve, according to considered values for the parameters, compressor behavior in dynamic state is visible. Produced curve from these three parameters should be located between surge lines of compressor map. According to produced model, if changes in the mass flow rate of air, which is according to the pressure ratio of compressor is investigated, centrifugal compressor map is obtained (Fig. 7).

![Fig. 7: Curve fitting according to obtained parameters of the compressor map](image)

According to requested electrical current of the fuel cell, mass flow rate of the output air in static compressor model changed based on the (8). Input pressure of compressor is equal to 1atm. In addition, output pressure of the compressor has been calculated according to output mass flow rate of the compressor and gas law equation. Hence, the amount of pressure ratio due to the input and output pressure is calculated. If, obtained results of the produced model are compared with the results of the model [8], the similarity of changes in the characteristics of the compressor map is obtained (Fig. 8). These characteristics are mass flow rate and pressure ratio.
Fig. 8: Changes in the characteristics of the compressor map for model and [8]

In the model of [8] same electrical current has been requested from the fuel cell complex. In Fig. 8, similarity of changes in the dynamic parameters of compressor (in the model and reference) indicates the desired results for the produced model. Additionally, during dynamic process and according to the input temperature and pressure ratio, changes in the output temperature of air can be investigated (due to extracted equations and produced model). As it has been shown in Fig. 9 (according to changes in the electrical current), the amount of compressor output temperature is increased or decreased. This parameter is used as an input parameter for cooler model of the fuel cell.

Fig. 9: Changes in the output temperature of compressor

In addition, output pressure and output mass flow rate of compressor are sent to sub models of fuel cell. In designed look-up tables (in Matlab/Simulink software), these two parameters are used to determining the compressor efficiency. Therefore, changes in the compressor efficiency in dynamic process have been shown in Fig. 10.

Fig. 10: changes in the compressor efficiency according to changes in the mass flow rate and pressure ratio

According to this paper, simulation of the centrifugal compressor has been performed in Matlab/Simulink software. Therefore, dynamic and comprehensive model has been presented for an air compressor. This model can be used as a subsystem in the fuel cell system. In this model, electric and thermodynamic parameters of air compressor are intercepted in dynamic status. This model can be used as the cathode air supply subsystem and can be added and developed for the fuel cell system.

4. CONCLUSIONS

In this paper, comprehensive model of the fuel cell air submitter has been presented. Simulation has been done with governed equations and modeling the relation of these equations. The main goals of this model are to increase the input pressure of the fuel cell and track changes in the output air parameters of compressor. Requested electrical current from the fuel cell is the input parameter for this model. The amounts of the electrical power have been calculated according to the static feed forward function and using compressor motor and static models.

In these two models, Difference in electrical powers determines the angular velocity of the compressor as an effective dynamic parameter. By using of this parameter (angular velocity) and extracted equations, electrical powers, mass flow rate and the compressor outlet pressure have been calculated. Hence, dynamic changes of the compressor characteristics according to the parameters of mass flow rate, outlet pressure and disturbance parameter of angular velocity have been shown in Fig. 8.

By comparing the results of this model and the model of [8], validation of the compressor map has been obtained. Additionally, changes in the main compressor characteristics have been located within surge lines. By using of the compressor characteristics (mass flow rate and outlet pressure) and extracted equations, changes in the outlet temperature (along the modeling process) have been obtained. In addition, according to embedded look-up tables and using of the parameters of compressor map, changes in the compressor efficiency has been obtained. Therefore, the comprehensive centrifugal compressor model is modeled that can be added to polymer fuel cell collection as a lateral system. With this system, input thermodynamic parameters to the fuel cell system are obtained. Main point is the communication between the requested electrical current and input thermodynamic parameters of cathode. These parameters are temperature, pressure and mass flow rate. Additionally, parasitic power of fuel cell (originated from the compressor) has been determined. Therefore, this model can predict the dynamic behavior of the compressor accurately and it can be used directly for any control purposes.

5. LIST OF SYMBOLS

<table>
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<th>parameter</th>
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<tr>
<td>d</td>
<td>m</td>
<td>diameter</td>
</tr>
<tr>
<td>J</td>
<td>kg.m²</td>
<td>inertia</td>
</tr>
</tbody>
</table>
Electric power \( P \) watt

pressure \( p \) pa

Mass flow rate \( Q \) kg/s

Gas universal constant \( R \) j/kg°C

temperature \( T \) °k

Blade’s tip speed \( U \) m/s

voltage \( V \) V

Mass flow rate \( W \) kg/s

Greek symbols:

<table>
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<tr>
<th>parameter</th>
<th>unit</th>
<th>symbol</th>
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<td>Specific heat rate</td>
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<td>( \gamma )</td>
</tr>
<tr>
<td>Efficiency</td>
<td>Non-dimensional</td>
<td>( \eta )</td>
</tr>
<tr>
<td>Temperature ratio</td>
<td>Non-dimensional</td>
<td>( \theta )</td>
</tr>
<tr>
<td>Density</td>
<td>kg/m³</td>
<td>( \rho )</td>
</tr>
<tr>
<td>Torque</td>
<td>N.M</td>
<td>( \tau )</td>
</tr>
<tr>
<td>Head coefficient</td>
<td>Non-dimensional</td>
<td>( \psi )</td>
</tr>
<tr>
<td>Angular velocity</td>
<td>rad/s</td>
<td>( \omega )</td>
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Subtitles

<table>
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<tr>
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<tr>
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</tr>
<tr>
<td>cp</td>
<td>compressor</td>
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</tr>
<tr>
<td>cm</td>
<td>Motor compressor</td>
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</tr>
<tr>
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6. ACKNOWLEDGEMENTS

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7. REFERENCES


