Multivariable Systems Model Reduction: A LMI Approach

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ABSTRACT:
This paper investigates a novel technique for Model Order Reduction (MOR) in Multi Input Multi Output (MIMO) systems. The problem of finding a Reduced Order Model (ROM) has been investigated by solving an H_∞ optimization problem as an equal convex optimization procedure. The reduced order model approximation derives out by simply solving a series of Linear Matrix Inequalities (LMIs). A comparative study have been made to illustrate the performance and efficiency of the proposed method. The important characteristics of the step response of both main system and its approximation model also have been considered in both time and frequency domain.

KEYWORDS: Model Order Reduction; Multi Input Multi Output (MIMO) System; Reduced Order Model (ROM); Convex Optimization; Linear Matrix Inequality (LMI).

1. INTRODUCTION

Model order reduction (MOR) in system theory defines as a technique for finding an approximation of higher order system by lower order models. Using such models make the analysis and simulation much easier and in many cases the stability of the reduced-order model is guaranteed if the original system be stable [1].Then this reduced-order model (ROM) can be evaluate with lower accuracy but in significantly less time. Many approaches were proposed in literature but the common approaches for model order reduction are Hankel norm based or projection-based reductions. Methods like proper orthogonal decomposition, proper generalized decomposition, balanced truncation or residualization or methods based on singular perturbations are presented in surveys for minimal MOR of state space systems. Some studies also analyze the problem in the frequency domain as well. It is obvious that truncated balance realization and Hankel norm reduction are good algorithms to find ROMs [2]. In this paper using H_∞ norm as criterion, we expand the idea of MOR to the multivariable systems which have multi inputs and multi outputs. Because of the higher dimension matrixes we are dealing with MIMO systems. It is critical to investigate this case in order to find a lower order model and reduce the amount of both time and computation. Recently linear matrix inequalities (LMIs) attracted a huge amount of attention in solving H_∞ problems due to the availability of the vast base of software's that can solve these kinds of problem more easy. In this paper considering a model reduction problem as a case of H_∞ design with a confined degree of the controller, we expand the idea of SISO-MOR to MIMO-MOR using inequalities like Bounded real lemma.

This paper is organized as follows: The problem statement formed in section II. In section III the new procedure for finding ROM has been presented. And in the penultimate section before the conclusion a numerical example is given to illustrate the validity of the proposed method.

2. PROBLEM STATEMENT

First, using external description for the common system with internal description as:

$$\dot{x} = Ax + Bu \quad y = Cx + Du \tag{1}$$

We get

$$G(s) = \frac{1}{D(s)}\begin{bmatrix} g_{11}(s) & \cdots & g_{1m}(s) \\ \vdots & \ddots & \vdots \\ g_{l1}(s) & \dots & g_{lm}(s) \end{bmatrix} \tag{2}$$

Where D(s) is the common denominator define as $D(s) = \prod_{l=1}^{n}(s+\lambda_n)$ which n is the number of poles.

The transfer function of the higher order system of order n represents as

$$G(s) = \frac{Y(s)}{R(s)} = \frac{a_1 s^{n-1} + a_2 s^{n-2} + \cdots +. a_n}{s^n + b_1 s^{n-1} + b_2 s^{n-2} +..+ b_n}$$
$$a_i \,\&\, b_i : const \quad i = 1,2,\ldots,n \qquad (3)$$

The problem is to find a reduced order model of less order than n, represented by r. Reduced order of the system (3) can be represented by (4). Note that this reduced order model should have all the important futures of the main system.

$$G_r(s) = \frac{Y_r(s)}{R_r(s)} = \frac{c_1 s^{r-1} + c_2 s^{r-2} + \cdots + c_r}{s^r + d_1 s^{r-1} + d_2 s^{r-2} +..+ d_r} \qquad (4)$$

The main purpose of the proposed LMI based method is to find constants c_r and d_r through solving H_∞ problem.

3. THE PROPOSED METHOD: H_∞OPTIMIZATION

The H_∞ problem which mostly applies to analyze the stability of system with uncertainties, has been entangled with the definition of bounded real lemma. Consider the problem of finding the H_∞-norm of system G defines as $G(s) = C(SI - A)^{-1}B + D$. The statement $\|G\|_\infty < \gamma$ is equal to fine a symmetric positive matrix P>0 in inequality below [3]:

$$\begin{bmatrix} PA + A^T P & PB & C^T \\ B^T P & -\gamma I & D^T \\ C & D & -\gamma I \end{bmatrix} < 0 \qquad (5)$$

This inequality is Bounded real lemma and can be easily solved using user friendly softwares and methods for solving LMIs.

If the reduced order model can be define as $\hat{G}(s) = \hat{C}(SI - \hat{A})^{-1}\hat{B} + \hat{D}$, the problem should be consider as an optimization problem subject to some matrix inequality constraints. $(\hat{A}, \hat{B}, \hat{C}, \hat{D})$ and P>0 can be find through solving a matrix inequality as

$$\begin{bmatrix} P\tilde{A} + \tilde{A}^T P & P\tilde{B} & \tilde{C}^T \\ \tilde{B}^T P & -\gamma I & \tilde{D}^T \\ \tilde{C} & \tilde{D} & -\gamma I \end{bmatrix} < 0 \qquad (6)$$

Where

$$\begin{bmatrix} \tilde{A} & \tilde{B} \\ \tilde{C} & \tilde{D} \end{bmatrix} = \begin{bmatrix} A & 0 & B \\ 0 & A & \hat{B} \\ C & -\hat{C} & D & -\hat{D} \end{bmatrix} \qquad (7)$$

Then by using algorithm below, it can provide the ROM of the original system [2]:

Input: internal description or state space matrices of $\hat{G}(s)$ which can be obtained from, for example Hankel order reduction or truncated balanced realization.

Main procedure: keeping $(\hat{A},\hat{B})$ constant, minimizing γ subject to (6) with respect to (P,$\hat{C}$,$\hat{D}$).

Output: the reduced order model (4) can be obtained through the results of optimization procedure.

4. SIMULATION

A numerical example has been simulated in this section in order to show the validity of the proposed method. Considering a two input two output, six order, time invariant continues system taking from the literature, the reduced order model is presented through solving optimization problem via Yalmip toolbox in MATLAB.

The step response of the original system and some of the ROMs studying recently in literature has been presented on Fig.1. Fig.2 shows how accurate our approximation of the proposed model takes the original system behavior. In order to demonstrate the accuracy between the approximations proposed ROM and the main system in frequency domain, the Nyquist diagram traced in Fig.3. There is also a comparison between the values of the step response data among the proposed method and some of the other techniques available in literature.

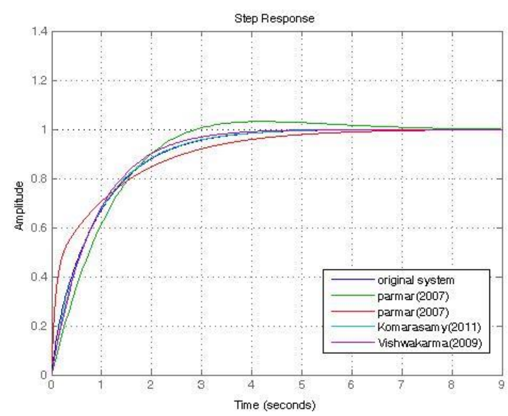

Fig. 1. Step response of the first channel (g_{11}) of the main complete order system and some of the reduced order models approximations studies in the literature.

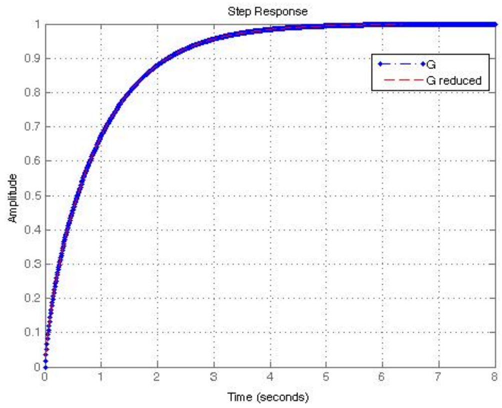

Fig. 2. Step response of the first channel (g_{11}) of the main complete order system and the reduced order model approximation derived by the proposed optimization technique.

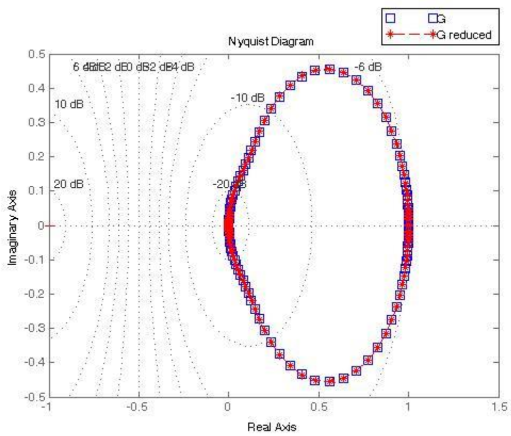

Fig. 3. Nyquist diagram of the first channel (g_{11}) of the main complete order system and the reduced order model approximation derived by the proposed optimization technique.

Table 1. Comparison between the step response characteristics of the first channel (g_{11}) of the main complete order system and the reduced order model approximation derived by the proposed optimization technique and in the literature.

Method	Rise time	Settling time	Peak	Peak time
Original system	2.1261	3.7942	0.9998	8.3722
Proposed optimization method	2.1263	3.7947	0.9998	8.3841
SE-GA [5]	1.8624	5.8176	1.0312	4.2314
FDA [6]	2.6393	5.1309	0.9981	8.7675
IPC[7]	2.0642	3.7541	0.9988	6.5389
PC[8]	1.9091	3.4067	1.0000	9.6310

5. CONCLUSION

A novel method of model order reduction for MIMO systems has been presented in this paper. Considering H_∞ norm optimization as a convex problem, a new procedure for obtaining the reduced order approximation of the main system has been submitted through solving a series of linear matrix inequalities. The efficiency of this technique was illustrated by a numerical example and also compared to the other recent work in literature. The step response data of our proposed ROM and the main system and also other works were compared in both time and frequency domain and it revealed that our technique lead to a more accurate and simpler approximation. Due to a recent progress in developing programs which solves analytical problems like LMIs more easy, this technique can be easily design and implement compare to the previous methods.

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