Perfect Modelling of Grounding Systems to Study Electromagnetic Transients Using Vector Fitting Method

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ABSTRACT:
As wind farms grow rapidly throughout the world, the grounding system and ground impedance of wind turbine towers under transients have got more important. As a matter of fact, an exact model to testify dynamic properties of grounding system should represent nonlinear effects of soil ionization and be frequency dependent. Some new researches conducted in this issue, all are based circuit theory which does not allow for accurate analysis of aforementioned behaviors. This paper has come to express a new approach to model any grounding system only by few RLC branches in wide frequency range. A new model and some simple formula that is recommended in this paper, represent all effects of transient phenomena of lightning strikes and all analysis have got more easier than other approach. By using this method, there’s no difference between those grounding system buried in cylindrical soils or conical one, multi-layer oil or not, humid soil or not and so on, they all are available for transient analysis. Then the outputs or the RLC branches are implemented in EMTP and get analyzed. So it’s possible to evaluate power systems under lightning strikes with presence of precise behavior of grounding systems.

KEYWORDS: Electromagnetic Transient, EMTP, Grounding System, Vector Fitting.

1. INTRODUCTION
Transient behaviors of grounding system directly impact on power network performance under short circuit faults or lightning strikes [1]. Approximately most of models presented in recent years were based on circuit theory or transmission line method, which do not have enough accuracy to express all transient behaviors of grounding systems [2]. In past, the researchers usually have tried to reach a model which can be more precise to show soil ionization under high frequency surges. But either they have got a low frequency model, or the models weren’t applicable with simulation package of power systems [3].

In circuit theory those capacitances which create because of image theory are generally neglected. The other disadvantage of circuit theory is that radius of lightning current under the earth change as time varying [4], but here this phenomena is neglected, too. These disadvantages could also be seen in transmission line theory [5].

Hence lack of a new approach to overcome all these weak modelling, cause the authors to recommend a new one. This approach is based on Vector Fitting method which could then implement all grounding system in a wide frequency range. All we need in this method is the frequency response of excitation signals applied to the grounding system just as a data file. Of course it’s nice to say that frequency response of each arbitrary circuit, can be reached from direct manners or by using 3-dementional finite element simulation tools [6]. Later, when a frequency response is available, VF method is hired to simulate those arbitrary systems only with some RLC branches.

2. TECHNICAL WORK PREPARATION
Consider the rational function approximation

\[
f(s) = \sum_{n=1}^{N} \frac{c_n}{s - a_n} + d + sh \tag{1}\]

The residues \(c_n\) and poles \(a_n\) are either real quantities or come in complex conjugate pairs, while \(d\) and \(h\) are real [21]. The problem at hand is to estimate all coefficients in (1) so that a least squares approximation of \(f(s)\) is obtained over a given frequency interval. We note that (1) is a nonlinear problem in terms of the unknowns, because the unknowns \(a_n\) appear in the denominator. Vector fitting solves the problem (1) sequentially as a linear problem in two stages, both times with known poles [7].
First we specify a set of starting poles $a'_n$ in (2), and multiply $f(s)$ with an unknown function $w(s)$. In addition we introduce a rational approximation for $w(s)$. This gives the augmented problem:

$$\begin{align*}
\begin{bmatrix}
    w(s) \\
    f(s)
\end{bmatrix}
&= \begin{bmatrix}
    \sum_{n=1}^{N} \frac{c_n}{s-a'_{n}} + d + sh \\
    \sum_{n=1}^{N} \frac{c'_n}{s-a'_{n}} + 1
\end{bmatrix}
\end{align*}$$

(2)

Note that in (2) the rational approximation for $w(s)$ has the same poles as the approximation for $w(s)f(s)$. Also, note that the ambiguity in the solution for $w(s)$ has been removed by forcing $w(s)$ to approach unity at very high frequencies.

Multiplying the second row in (2) with $f(s)$ yields the following relation [7]:

$$\sum_{n=1}^{N} \frac{c_n}{s-a'_{n}} + d + sh = \left(\sum_{n=1}^{N} \frac{c'_n}{s-a'_{n}} + 1\right)f(s)$$

(3)

Or

$$(wf)_{fit}(s) = w_{fit}(s)f(s)$$

(4)

Equation (3) is linear in its unknowns $c'_{n}$, $h$, $d$ and $c_n$. Writing (4) for several frequency points gives the over determined linear problem $Ax = b$, Where the unknowns are in the solution vector $x$. Equation (4) is solved as a least squares problem. Details about the formulation of the linear equations is shown in [8].

A rational function approximation for $f(s)$ can now be readily obtained from (3). This becomes evident if each sum of partial fractions in (3) is written as a fraction:

$$(wf)_{fit}(s) = h \frac{\prod_{n=1}^{N+1}(s-z)}{\prod_{n=1}^{N}(s-a'_{n})}$$

(5)

From (5) we get

$$w_{fit}(s) = \frac{\prod_{n=1}^{N+1}(s-z')}{\prod_{n=1}^{N}(s-a'_{n})}$$

(6)

Equation (6) shows that the poles of $f(s)$ become equal to the zeros of $w_{fit}(s)$! (Note that the starting poles cancel in the division process because we use the same starting poles for $(wf)_{fit}(s)$ and for $w_{fit}(s)$. Thus, by calculating the zeros of $w_{fit}(s)$ we get a good set of poles for fitting the original function $f(s)$ . The calculation of zeros from the representation by partial fractions (1) is straightforward, as shown in [8].

3. CIRCUIT SYNTHESIS IMPLEMENTATION

If frequency response of any circuits is available in form of equation (1), then each term of that transfer function could be modeled as a simple RLC branch. Hence these branches should be connected to each other in series and finally total grounding system impedance will be implemented in EMTP at wide frequency range.

In equation (1), coefficients $c_n$ and $a_n$ can be real or complex, but $d$ and $h$ must be real if they exist. The amount of each branch could be calculated as follow:

1) Coefficients $d$ and $h$ respectively determine real resistance and inductance of grounding system. So we have:

$$d = R_0 , \quad h = L_0$$

(7)

2) For positive $c_n$, the transfer function will appear as (8) that poles and zeros are both real which approximated to:

$$Z_i(s) = \frac{1}{C_i} \frac{1}{s + \frac{1}{R_1 C_i}}$$

(8)

Where

$$C_i = \frac{1}{c_i} & \quad R_i = \frac{a_i}{c_i}$$

(9)

3) If coefficients $c_n$ in any transfer function is negative, it should be considered as (10), Hence poles and zeros are both real and approximated as follow:

$$Z_j(s) = \frac{s \times R_2}{s + \frac{R_2}{L_1}}$$

(10)

$$Z_j(s) = R_2 + \frac{\frac{R_2^2}{L_1}}{s + \frac{R_2}{L_1}}$$

(11)

$$L_1 = -\frac{C_j}{a_j} & \quad R_2 = -\frac{C_j}{a_j}$$

(12)

Then $R_0$ may has got a new value as:
4 new = d old - R_2 \tag{13}

4) If poles are conjugate complex, then transfer function may appear as (14):

\[ Z_k(s) = \frac{s \left( \frac{1}{L_2} \right) + R_3}{s^2 + s \left( \frac{R_3 + R_4}{L_2} \right) + \frac{R_3 + R_4}{L_2 C_2}} \tag{14} \]

And this term is appeared while it could be separated to (15):

\[ Z_k(s) = \frac{c_r + j a_r}{s + (a_r + j a_i)} + \frac{c_r - j a_r}{s + (a_r - j a_i)} \tag{15} \]

Now the branch components are calculated as:

\[ C_2 = \frac{1}{2c_r} \tag{16} \]

\[ L_2 = \frac{2c_i^2}{a_i^2 c_i^2 + a_i^2 c_r^2} \tag{17} \]

\[ R_3 = \frac{2c_r^2(a_i c_i + a_r c_r)}{a_i^2 c_i^2 + a_i^2 c_r^2} \tag{18} \]

\[ R_4 = -\frac{2c_r^2}{a_r c_i - a_r c_r} \tag{19} \]

Finally these impedances are gathered together in series which is depicted in Fig. 1.

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**4. SIMULATION AND ANALYSIS**

Imagine that a frequency response of any circuit is available as a data file, so while VF method runs via MATLAB software, the outputs represent all coefficients \( a_n, c_n, h \) and \( d \), which are all known. Then the data obtained from abovementioned grounding system runs by this method. The aforementioned approach will be implemented and evaluated in two examples.

\textit{a) Example (I)}

The data used in [9] is hired to evaluate via method presented in this paper. After 10 iterations, the outputs are obtained with \( 9.911 \times 10^{-9} \) rms error in about 2.971 seconds run time. The impedance magnitude and phase angle plots are depicted in Fig. 2 and Fig. 3 respectively. Blue plot shows the basic data and red plot represent the results obtained from VF method. Results are shown in appendix A.

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**Fig 2.** Magnitude of grounding impedance; Comparison between data [9] and VF results

**Fig 3.** Phase angle of grounding impedance; Comparison between data [9] and VF results
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So the equivalent circuit (by 4th order approximation function) implemented in EMTP is shown in Fig. 4.

![Fig. 4. Equivalent circuit of grounding system implemented in EMTP](image)

Then results obtained from electromagnetic simulations in EMTP could be seen in Fig. 5 and 6 respectively.

![Fig. 5. Magnitude of grounding impedance implemented in EMTP](image)

![Fig. 6. Phase angle of grounding impedance implemented in EMTP](image)

As it’s depicted in above pictures, the plots of grounding system behaviors are very close to reconcile to each other.

b) Example (II)

For more validating of this approach, the data used in [10] is hired to evaluate via method presented in this paper. After 10 iterations, the outputs are obtained with $1.696 \times 10^{-8}$ rms error in about 3.102 seconds run time. After 10 iterations the coefficients are calculated as shown in appendix B.

The impedance magnitude and phase angle plots, are shown in Fig. 7 and Fig. 8 respectively. Blue plots are main data and red plots represent outputs which are calculated by VF method.

![Fig. 7. Magnitude of grounding impedance; Comparison between data [10] and VF results](image)

![Fig. 8. Phase angle of grounding impedance; Comparison between data [10] and VF results](image)

The rms error of these calculations is 2.6754 percent which is depicted in magnitude plot by green color. Then
the equivalent circuit (by 5th order approximation function) implemented in EMTP is shown in Fig. 9.

![Fig. 9. Equivalent circuit of grounding system implemented in EMTP](image)

The results shown in Fig. 10 and Fig. 11 include soil ionization effects, sort of soil which grounding system buried in, and high frequency behavior. These outputs represent that all grounding systems are easily possible to be modeled and utilized in evaluation package of power systems such as DigSilent software or EMTP package and etc.

![Fig. 5. Magnitude of grounding impedance implemented in EMTP](image)

![Fig. 6. Magnitude of grounding impedance implemented in EMTP](image)

5. CONCLUSION

When lightning strikes collides to ground as a surge wave, grounding system is excited that some high frequency components of current are created. Those components under the ground cause soil ionization that is the main reason which make transient properties of grounding system nonlinear. Thus if grounding system is buried in cylindrical or conical kind of soil, modelling of its electrodes in order to evaluate power systems in simulation packages are approximately impossible.

This paper presents a novel approach to model and simulate all these events caused by lightning strikes. By this method, it’s possible to simulate each grounding system only with some simple RLC branches which has a property equation of low order in large frequency range. This approach also has higher accuracy rather than other methods mentioned in several references. For instance, in most researches soil ionization and delay prediction of waves propagation are not considered. In other hand, by using this method, it’s easy to analyze even complex grounding system via simple processor. While aforementioned methods need more parallel processor to study grounding properties. Because they usually use many voltage sources and current sources which are dependent to previous instances.

Finally the outputs of presented method are compared and validated with results obtained from antenna theory approach hired by references.

REFERENCES


6. APPENDIX A.
Results VF method for grounding system evoked from [9] are shown as follow:

*** Vector Fitting is starting … ***
Creating frequency response $f(s)$…
Resulting state space model:

$$
A = 
\begin{pmatrix}
1.0e+02 \\
0.0496 + 0.0000i & 0.0000 + 0.0000i \\
0.0000 + 0.0000i & -1.0000 + 5.0000i \\
0.0000 + 0.0000i & 0.0000 + 0.0000i \\
\end{pmatrix}
\begin{pmatrix}
0.0000 + 0.0000i & 0.0000 + 0.0000i \\
0.0000 + 0.0000i & -1.1200 + 0.0000i \\
0.0000 + 0.0000i & 0.0000 + 0.0000i \\
0.0000 + 0.0000i & 0.0000 + 0.0000i \\
\end{pmatrix}
$$

Columns 1 through 2
-0.0496 + 0.0000i 0.0000 + 0.0000i
0.0000 + 0.0000i -1.0000 + 5.0000i
0.0000 + 0.0000i 0.0000 + 0.0000i

Column 3
0.0000 + 0.0000i
0.0000 + 0.0000i
-1.0000 - 5.0000i

B =
1
1
1
1

C =

Columns 1 through 2
0.0001 + 0.0000i 15.0000 +20.0000i

Column 3
15.0000 -20.0000i

D = 0.5000

*** DONE ***

Then the transfer function is approximated as follows:

$$
f(s) = C(sI - A)^{-1}B + D + sE
$$

7. APPENDIX B.
Results VF method for grounding system evoked from [10] are shown as follow:

*** Vector Fitting is starting … ***
I. Creating frequency response $f(s)$…
II. Resulting state space model:

$$
A = 
\begin{pmatrix}
1.0e+03 \\
0.0025 + 0.0000i & 0.0000 + 0.0000i \\
0.0000 + 0.0000i & -1.1200 + 0.0000i \\
0.0000 + 0.0000i & 0.0000 + 0.0000i \\
0.0000 + 0.0000i & 0.0000 + 0.0000i \\
\end{pmatrix}
\begin{pmatrix}
0.0000 + 0.0000i & 0.0000 + 0.0000i \\
0.0000 + 0.0000i & -1.1200 + 0.0000i \\
0.0000 + 0.0000i & 0.0000 + 0.0000i \\
0.0000 + 0.0000i & 0.0000 + 0.0000i \\
\end{pmatrix}
$$

Columns 1 through 2
-0.0025 + 0.0000i 0.0000 + 0.0000i
0.0000 + 0.0000i -1.1200 + 0.0000i
0.0000 + 0.0000i 0.0000 + 0.0000i
0.0000 + 0.0000i 0.0000 + 0.0000i

Columns 3 through 4
0.0000 + 0.0000i 0.0000 + 0.0000i
0.0000 + 0.0000i 0.0000 + 0.0000i
0.0000 + 0.0000i 0.0000 + 0.0000i
-0.1000 + 0.5000i 0.0000 + 0.0000i
0.0000 + 0.0000i -0.1000 - 0.5000i

B =
1
1
1
1

C =

Columns 1 through 2
0.0001 + 0.0000i 15.0000 +20.0000i
Columns 3 through 4
30.0000 -40.0000i 30.0000 +40.0000i

D =
2.0000

E = 1.0000e-05

rmse = 1.6960e-08